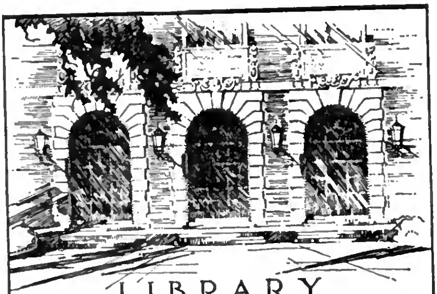

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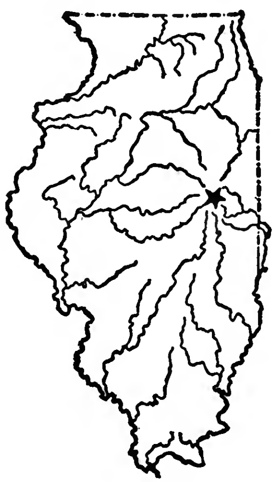
BULLETIN No. 288

PERSISTENCY OF LACTATION IN DAIRY COWS

A Preliminary Study of Certain Guernsey and Holstein Records

By W. L. Gaines



URBANA, ILLINOIS, APRIL, 1927

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PERSISTENCY OF LACTATION IN DAIRY COWS

A Preliminary Study of Certain Guernsey and Holstein Records[a]

By W. L. Gaines, Chief in Milk Production

INTRODUCTION

The term persistency of lactation is used to refer to the degree with which the rate of milk secretion is maintained as lactation advances. Cows ordinarily reach their highest rate of milk secretion in any one lactation period, shortly after calving. Following the flush of lactation, the rate of milk secretion declines more or less rapidly until the cow goes dry naturally or is dried up artificially by discontinuing milking.

It is clear that, other things being equal, the more persistent a cow is (that is, the less rapidly she declines in rate of milk secretion), the more milk she will produce in a year's time. It is commonly stated that the dairy breeds produce more milk per year than the beef breeds or unimproved cattle partly because they are more persistent milkers. As between the cows of the dairy breeds it is recognized, in turn, that some are more persistent than others. In the official testing of the dairy breeds, the short-time test, such as the 7-day test, has been very adversely criticised because it does not depend upon and does not measure (it is said) the quality of persistency. In general persistency of lactation is counted as a very important factor in the yearly yield and economical production of milk. As a matter of fact, however, our knowledge of persistency of lactation as a character of dairy cows is very limited so far as exact quantitative analysis is concerned.

The American Guernsey Cattle Club has published about 15,000 yearly records of Guernsey cows in detail by calendar months. The Holstein-Friesian Association of America has published about 1,500 7-day records made at least eight months after calving, together with in each case a 7-day record made early in the same lactation. These breed records afford data of great value for quantitative study of persistency, and the present paper is the outcome of an attempt to analyze certain of them from the standpoint of the persistency of lactation shown by the individual records.

[a]Submitted for publication June 29, 1926.

The Guernsey records used (1,676) were limited to original entries published in Vols. 33 and 34 and No. 1 of Vol. 35 of the Herd Register. Only 365-day records which represented a single lactation and in which conception did not recur within 6 months after calving were used. This selection was intended to eliminate any disturbing influence of pregnancy on persistency. Also, only those records were used in which not more than 75 days elapsed from calving to the middle of the first full calendar month of the record, in order that the records should be fairly comparable with respect to the time after calving at which they started.

The Holstein records used (1,395) constitute all the eight-months-after-calving records published in Vols. 24 to 31 of the Advanced Register Year Book, with certain few exceptions noted later. The volumes named include nearly all the records of this class which have been published.

THE PROBLEM

The problem with respect to the Guernsey records may be presented by considering the two records reproduced in Fig. 1. Mere inspection of these records is sufficient to show that one of the cows (10233) was a very persistent milker. Ignoring the record for June because it is for only part of the month, it is apparent that there is only a slight decrease in the monthly milk yields with advance in lactation. The corresponding fat yields show a tendency to increase rather than to decrease. The other cow (10372) shows by her record a tendency to decrease very rapidly in monthly milk and fat yield with advance in lactation. She would be classed as a very non-persistent milker.

It is not sufficient for quantitative study to say that one cow is persistent and another is not. The problem, first of all, is to derive a numerical value for persistency as shown by the individual record. This in itself is somewhat difficult and offers opportunity for difference of opinion as to what may be accepted as the best value for the purpose.

The two records given in Fig. 1 represent opposite and rather extreme cases of persistency. What we should like to have is a completely representative picture of the breed as a whole with respect to the persistency character. Having given an acceptable numerical value of persistency for each record of a suitable group of cows, such a representative picture may be obtained by the usual statistical methods. The character may then be studied after the same fashion as milk yield, fat yield, or fat percentage have already been studied by various investigators, notably at the Maine Station (cf. Gowen[8]).

Some of the questions relating to persistency are: the form of its frequency distribution curve; its relation to the age of the cow; its

relation to yearly yield; its relation to the rate of yield at the flush of lactation; its relation to genetic factors; its relation to environmental factors. In short, what are the attributes of persistency of lactation as a characteristic of dairy cows as shown by the advanced registry records?

10233 CHOICE OF ELMWOOD 51908

Sire Sir Ponto of Elmwood 25492.
Sire Katonah's Mack 17252.
Dam Faithful of Elmwood 39350.
Dam Flossie of Elmwood 35949.
Sire Rex of Eastside 12763 A. R.
Dam Dolly Varden of Eastside 20756.
Breeder M. E. Gifford, Sherman, N. Y.
Owner George S. Love, Waukesha, Wis.
Born Dec. 9, 1913. *Calved* June 4, 1920.
Served Apr. 4, 1921.
Requirement for admission: 360.00 lbs. fat.
Supervised by Wisconsin Station.
3 milkings daily.

Official Year's Record, Class A

	Milk	Butter Fat	
1920	lbs.	%	lbs.
June 7,	919.2	4.28	39.34
July,	1264.5	4.34	51.88
Aug.,	1181.9	4.36	51.53
Sept.,	1179.6	4.40	51.90
Oct.,	1236.1	4.60	56.86
Nov.,	1181.6	4.47	52.82
Dec.,	1212.0	4.51	54.66
Jan., 1921	1205.4	4.75	57.26
Feb.,	1077.8	5.05	54.43
Mar.,	1115.8	4.81	53.67
Apr.,	1109.1	4.78	53.01
May,	1113.5	5.45	60.69
June 6,	223.8	5.45	12.20
Total	**14020.3**	**4.66**	**653.25**

10372 DORA OF ELMENDORF 56322

Sire Imp. Lord Mar V. 18961 A. R.
Sire Imp. Lord Mar 14359 A. R.
Dam Imp. Countess I. of Les Nouettes 36184.
Dam Imp. Dora of the Vrangue VI. 36193.
Sire Imp. Galaxy's Sequel 16904 A. R.
Dam Dora of the Vrangue R.G.A.S. 5572 P.S.
Breeder J. B. Haggin, Lexington, Ky.
Owner R. M. Cooper, Jr., Wisacky, S. C.
Born Nov. 15, 1913. *Calved* Dec. 16, 1919.
Served Oct. 8, 1920.
Requirement for admission: 360.00 lbs. fat.
Supervised by South Carolina Station.
3 milkings daily.

Official Year's Record, Class A

	Milk	Butter Fat	
1919	lbs.	%	lbs.
Dec. 22,	497.2	4.56	22.67
Jan., 1920	1609.4	4.56	73.39
Feb.,	1244.1	4.96	61.71
Mar.,	1144.8	4.55	52.09
Apr.,	1022.9	4.52	46.24
May,	892.0	4.98	44.42
June,	656.8	4.57	30.02
July,	427.2	4.56	19.48
Aug.,	319.9	5.19	16.60
Sept.,	309.8	4.59	14.22
Oct.,	288.9	5.40	15.60
Nov.,	223.7	4.67	10.45
Dec. 20,	155.9	4.63	7.22
Total	**8792.6**	**4.71**	**414.11**

FIG. 1.—PHOTOGRAPHIC REPRODUCTION OF TWO RECORDS FROM VOL. 35 OF THE GUERNSEY HERD REGISTER

These two records are chosen as striking examples of the differences in persistency shown by the published records. They serve also to illustrate the various data given in connection with each record. The leaves of the Herd Register were removed and backed by sheets of gummed paper board and then cut into cards, one to each individual record. Derived data for each record were recorded on its card. The records were thus brought into convenient form for manipulation for statistical purposes.

As indicated, the first step is to derive a numerical expression for persistency, and a method of doing this we may consider in connection with the two records given in Fig. 1 as examples.

THE METHOD

The expression of persistency of lactation used in the present treatment is essentially an old one. Sturtevant[18] studying the average yield

of a herd of cows, used the method of expressing the decrease in yield of milk from month to month as a percentage of the yield of the previous month, and found that as thus expressed the decrease tended to be constant. This expression or its complement, the expression of the milk yield for any month as a percentage of the yield of the preceding month, has been used by various investigators, and gives a numerical measure of persistency, the meaning of which is readily grasped.

Brody *et al*[1] have put the expression into the closely related form of an exponential equation. While this form may not be quite so easily understood as Sturtevant's it is on the whole a more reasonable expression, giving concisely both the rate of decrease and the rate of yield. From experience[6] with the use of the exponential expression and its application to the group performance shown by Guernsey records, it seemed to the writer that it might be applied to individual records to give a reasonable numerical expression of persistency.

In theory the idea is that the *rate* of milk secretion is continuously decreasing with advance in lactation in accordance with the equation:

$$\frac{dy}{dt} = ae^{-kt} \tag{1}$$

in which y = yield in pounds; t = time in months from calving; $\frac{dy}{dt}$ is the rate of yield in pounds per month; e (= 2.71828) is the base of natural logarithms; a is the theoretical initial rate of yield; and k is the rate of change per month in the rate of yield[a]. In this equation k is the factor which is used as a measure of persistency. The minus sign means that the rate of yield is decreasing and k taken as a positive value is a measure of this decrease, differing from Sturtevant's expression of the percentage decrease per month in that it is reckoned as occuring continuously instead of at monthly intervals. The rate of decrease is thus referred to the rate of yield at the immediate time rather than to the rate of yield a month preceding.

We may consider the application of equation (1) to the records of Fig. 1. In the first place it is evident that the records are based on the calendar month, which varies from 28 to 31 days, and it is necessary to make allowance for this fact. This is conveniently done by reducing the monthly yields to an average yield per day.

It is also apparent from Fig. 1 that there is a tendency for the percentage of fat in the milk to increase from month to month with advance in lactation. This is characteristic of the effect of advance in

[a]The mathematical relations involved in the derivation and use of the equation are given in more detail in Bulletin 272[6].

lactation, as has been shown in considerable detail by Turner.[19] Altho it happens in the two records shown in Fig. 1 that the average fat percentage is very similar, as between a larger number of individuals there inevitably will be marked differences in this particular which should be taken into account. These individual and monthly differences in fat percentage and correlated percentage of other solids are taken into account by dealing with estimated energy yields in terms of 4-percent milk. The estimation is made by the formula[5] F.C.M. = $.4M + 15F$, where F.C.M. is "fat-corrected milk," M is milk; and F is fat, all in pounds. One pound F.C.M. = one pound 4-percent milk = 331 large calories.

Table 1 gives the average daily F.C.M. values for the eleven full calendar-month records of the two cows in Fig. 1. The record in this form shows more clearly than does Fig. 1 the marked difference in persistency of the two cows. The values are given graphically in Fig. 2. The next step in the work is to apply equation (1) to the data of Table 1. This cannot be done directly; the equation in form for application may be written,

$$y_d = Ae^{-kt} \tag{2}$$

in which y_d is the yield for a month expressed in terms of pounds of F.C.M. per day. Time, t, is reckoned to the middle of the month and $\frac{365A}{12} = a\frac{e^{.5k} - e^{-.5k}}{k}$. Equation (2) may be converted to a linear form by taking logarithms on both sides, giving,

$$\log_{10} y_d = \log_{10} A - kt \log_{10} e \tag{3}$$

By the use of equation (3), values for A and k are readily derived from the observed values of Table 1. Using the method of least squares, the equation for No. 10233 becomes $y_d = 41.854\, e^{.001712t}$, and for No. 10372, $y_d = 74.525\, e^{-.202893t}$. The corresponding curves are given in Fig. 2. If we wish to use equation (1) for cow No. 10372 we have $\frac{dy}{dt} = \frac{365}{12} \times 74.401e^{-.202893t}$, that is, $a = \frac{365}{12} \times .99834A$. In the case of No. 10233 the A constant is affected only slightly in the third decimal by the same transformation. The k constant is not affected in passing to the form of equation (1). For practical purposes, therefore, in the present data, we may assume $\frac{365}{12} \times y_d = \frac{dy}{dt}$ and $\frac{365}{12} \times A = a$.

We have thus expressed the rate of yield or lactation curve in terms of two constants, A and k, the values of which have been determined.

TABLE 1.—AVERAGE DAILY F.C.M. YIELDS OF TWO COWS IN GUERNSEY ADVANCED REGISTER
(Records for 11 full calendar months)

Full calendar month	Average daily F.C.M. yield (pounds)	
	No. 10233	No. 10372
First	42.9	56.3
Second	40.2	49.1
Third	41.7	40.0
Fourth	43.5	36.8
Fifth	42.2	33.0
Sixth	42.1	23.8
Seventh	43.3	14.9
Eighth	44.6	12.2
Ninth	40.4	11.2
Tenth	41.3	11.3
Eleventh	43.7	8.2

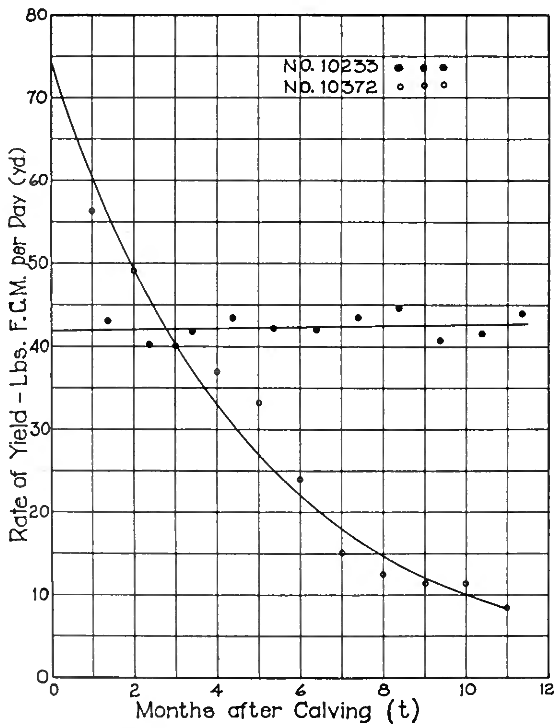


FIG. 2.—EXTREMES IN PERSISTENCY (TABLE 1) AND FITTED CURVES

Equations of curves, $y_d = Ae^{-kt}$: No. 10233, $y_d = 41.9e^{.002t}$; No. 10372, $y_d = 74.5e^{-.203t}$. Similar curves have been derived, one for each of the 1,534 Guernsey records and 1,395 Holstein records studied. The individual curves are studied with reference to the A and k constants, particularly the k constant, which is used as a measure of persistency.

While it is apparent from inspection of the records as first given in Fig. 1 that there is a great difference in the persistency of the two cows, we have now a numerical basis for the comparison of this difference in the factor k. No. 10372 is given a value of .203 and No. 10233, a value of $-.002$. Similar values may be determined for other cows from their records.

It is convenient in the recording and in the subsequent handling of the values of k to multiply by 1000. This product, $k \times 10^3$, may be considered as expressing the rate of decrease in per mille per month.

In the case of No. 10233 the rate of energy yield with advance in lactation is not decreasing but *increasing.* Since we are treating k as a positive value representing the rate of *decrease*, we in this case must give the value of k a minus sign. It would be absurd to suppose that this increasing rate of yield would continue indefinitely, especially in accordance with the equation, but the fact of the record is that the rate of yield is increasing and this is simply recognized as a negative decrease. We shall see later that an appreciable proportion of the records studied show this same feature.

While we are concerned primarily with persistency of lactation, that is, the k constant, we have to consider also the significance of the A constant. By the equation, $t = 0$ at calving, and accordingly at that time $Ae^{-kt} = A$; that is, the initial rate of yield is the value of A. It will be apparent that A is thus a hypothetical quantity since the full rate of yield is not realized immediately at calving. In the case of No. 10233 the rate of yield, A, is actually realized at a time later than calving. In the case of No. 10372 the rate of yield, A, is never realized. No. 10372 is, of cour e, an extreme case. In general, the larger the value of k the more the value of A exceeds the maximum realized rate of yield.[a] But nevertheless it will be apparent that A is closely related to the maximum realized rate of milk secretion.

Figs. 4 and 5 are intended to illustrate further the properties of the A and k constants of the equation.

The determination of the A and k constants has been carried out by the use of graphic methods illustrated and described in Fig. 3. The algebraic and geometric processes illustrated in Fig. 3 are all straightforward but there is an element of weakness, from the standpoint of mathematical precision, in fitting the straight line by the eye. In many cases the plotted observations are quite irregular and the fitting of the straight line requires the exercise of judgment. The determinations of A and k lack absolute precision on this account. Some records were so irregular that it was felt that extraneous factors must be so affecting the lactation

[a]This fault of the expression, if it is a fault, might easily be remedied by dealing with the lactation curve starting some time after calving, for example, by taking the time origin at one month after calving.

curve that for the purpose of the present study more trustworthy results would be secured by not including such records. On this ground, out of 1,676 records considered, 142, or 8.5 percent, were excluded.

In fitting the straight line it was attempted to make its slope correspond to the tendency of the plotted observations; and to adjust its level to the average level of the observations. As a check on the ac-

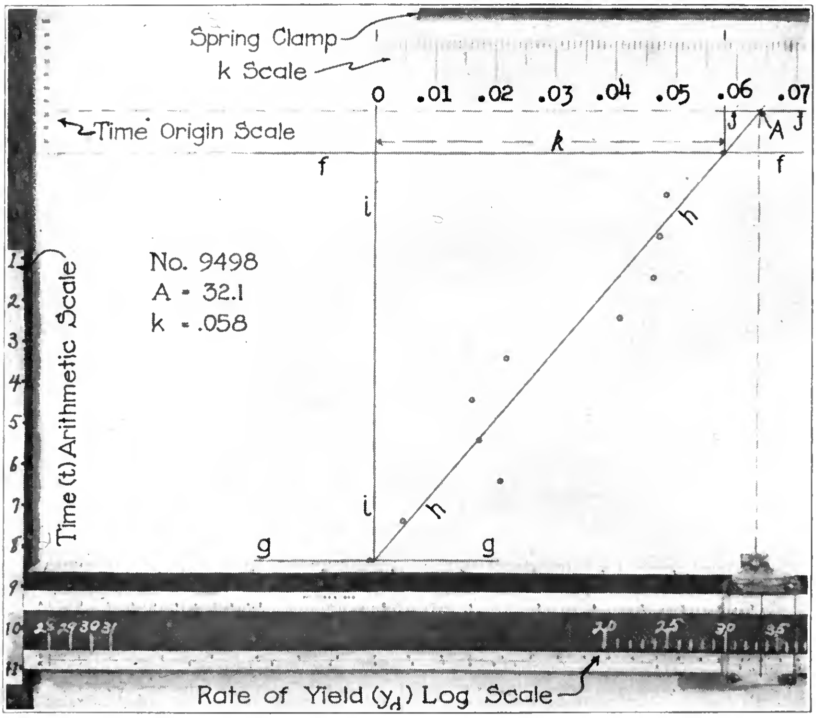


FIG. 3.—ILLUSTRATING THE METHOD OF DETERMINING THE A AND k CONSTANTS OF THE LACTATION CURVE

The equation (2) of the curve is $y_d = Ae^{-kt}$, in which y_d is the yield for a month expressed in terms of F.C.M. per day; and t is time in months from calving as origin and reckoned to the middle of the month.

Reference to Fig. 1 shows that there are 11 full calendar months in the cow's record and it is to these 11 observed values that the curve for each cow has to be fitted. The record (see Fig. 1) gives directly the pounds of milk and the percentage of fat for each month. The number of days in the month varies from 28 to 31. The average daily F.C.M. yield is computed by the use of a 500 mm. slide rule provided with a specially graduated slide. Consider a 30-day month and let M = recorded milk yield (lbs.) and f the corresponding fat percentage, then the observed $y_d = \frac{.4M + .15Mf}{30} = M\ (.01333 + .005f)$. A unity graduation (marked 30) is made on the slide, and with this coinciding with the unity graduation on scale D of the rule a graduation (marked 2.0) for 2.0 percent fat is made on the slide opposite 2333

(= .01333 + 2 x .005) of scale D. Similarly fat-percentage graduations are made by intervals of .1 to 9.0 percent.

If the milk yield for a 30-day month is 1,071 pounds and the fat percentage 3.33, the slide is set as in the illustration with the "30 unity" opposite 1071 on scale D, and the runner is set at 3.33 on the fat-percentage scale of the slide. Under the runner the value 321 is read on scale D, and the observed $y_d = 32.1$. To take care of months of 28, 29, or 31 days, it is clear that it is only necessary to provide appropriate unity graduations on the slide.

Equation (2) is transformed to the linear logarithmic expression:

$$\log_{10} y_d = \log_{10} A - .4343kt \tag{3}$$

In fitting equation (3) graphically we have to plot y_d on a log scale and time on an arithmetic scale. (The effect of the variation in the length of the calendar months is negligible here.) For the purpose of this plotting there are mounted on a drawing board two parallel guide bars, spaced to accommodate the length of the slide rule between them. These bars are each provided with 11 equally spaced notches in opposite pairs. The slide rule is provided at either end with a small lug to engage these notches. The arithmetic time spacing is thus provided for. The logarithmic y_d spacing is provided for by the construction of the slide rule itself.

The time origin (calving) is, of course, variable with respect to the month, being anywhere from 15 to 75 days preceding the middle of the first full calendar month. To locate this origin a scale is fixed alongside the extension of the guide bars, graduated in days, 30.5 graduations equaling one space on the guide bars. Zero of this scale corresponds in position to the upper edge of the rule when the latter is placed in notch 1, for the first full calendar month.

The description may be completed by following thru a determination. In operation the plotting paper, of plain white letter size, is properly placed according to the level of production of the record under consideration, and held in position by the spring clamp shown at the top of the illustration. The rule is placed in notch 1 and the line ff is drawn along its upper edge. The value of y_d for the first full calendar month is computed as above outlined and plotted by a small circle made thru the eye of the pointer on the runner. Graphically this is $\log_{10} y_d$, unity of the scale D representing 10 pounds. (It is not necessary actually to read the value of y_d.)

The rule is then moved to notch 2 and y_d for the second month plotted; and so on to the eleventh month. With the rule still in notch 11 the line gg is drawn along its upper edge.

The rule is then removed, and with a celluloid triangle used as a straight edge the line hh is drawn to fit the plotted values by inspection. The line ii is then drawn at right angles to gg thru the point of intersection of lines gg and hh. This is accomplished by adjusting the triangle against a guide strip (not shown) at the bottom of the apparatus.

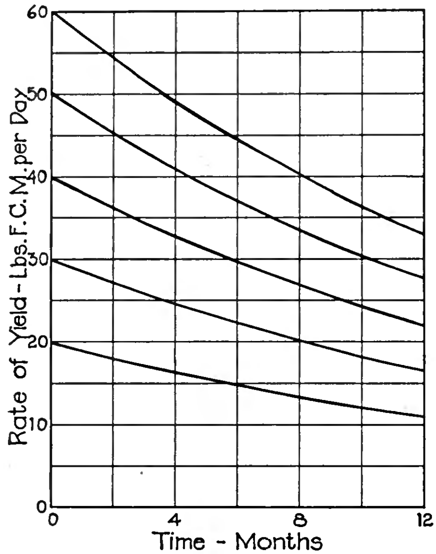
The rule is then adjusted on the time origin scale according to the number of days from calving to the middle of the first full month (32 days in the example), and the corresponding line jj drawn. Without moving the rule the runner is so set that the eye of the pointer coincides with the intersection of lines jj and hh. The corresponding reading on scale D gives the value of A, 32.1 in the example.

The value of k is proportional to the distance between the points of intersection of lines hh and ii with line ff. This distance corresponds to 10 units of time and therefore represents $4.343k$ in terms of the spacing of scale D of the slide rule. This scale is based on 500 mm. between the unity graduations. Accordingly a distance of 2171 mm. (4.343×500) corresponds to a value of $k = 1$ and 217.1 mm. corresponds to a value of $k = .1$. A suitable k scale is easily prepared being simply a uniformly graduated decimal scale with a length of 217.1 mm. between the zero and the .1 graduations. The value of k is obtained by placing the k scale alongside of line ff with the zero on the line ii, and reading the scale at the point of intersection of lines ff and hh. In the example given $k = .058$.

This completes the solution of the particular record. The device takes care of values of y_d from 10 to 100. In the few cases where y_d values of less than 10 are encountered, they are handled by multiplying the milk yields by a suitable factor and correcting the reading for A accordingly. Where unusually large values of k are involved, as in the case of No. 10372, Fig. 1, the necessary plotting range is secured by the use of two regular sheets properly adjusted.

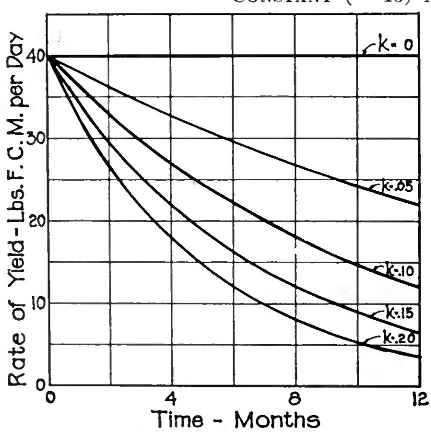
Only the heavier solid lines are drawn, in practice, the lighter broken lines being shown for the sake of explanation.

FIG. 4.—FORM OF THE THEORETICAL LACTATION CURVE WITH k CONSTANT (= 05) AND A VARIABLE



Cows represented by these curves are given the same persistency value, and it will be clear that this does not mean they have the same rate of absolute decrease in rate of yield. If the ordinates of these curves were plotted on a logarithmic scale, the curves would appear as straight parallel lines, that is, all with the same slope. Constant persistency then means that the rate of decrease bears a constant ratio to the rate of yield; or the slope of the lactation curve expressed in logarithms is constant. It may be noted that the areas under the curves are proportional to the values of A, or any other ordinate. Curves of the same persistency result in yearly yields proportional to their A's.

FIG. 5.—FORM OF THE THEORETICAL LACTATION CURVE WITH A CONSTANT (= 40) AND k VARIABLE



If the ordinates of these curves were plotted on a logarithmic scale, the curves would appear as straight lines and with slopes proportional to their k's. The measure of persistency k, is therefore based directly on the slope of the lactation curve when expressed in terms of logarithms. The areas under these curves would be inversely proportional to their k's (where $k > 0$) if the time considered were greatly extended. For the arbitrary period of a year the relation between k and yield is not simple, the yield being proportional to $\frac{1 - e^{-12k}}{k}$, with A constant. However, within the range of the most frequent values of k, equal changes in k result in approximately equal inverse changes in yield.

curacy of the fitting, a number of records (205) were selected at random and the theoretical twelve months' yields computed from the corresponding equations. In determining the theoretical yield due regard

was had for the time after calving at which the record started in order that the computed yield should be comparable with the observed yield. The yield as thus computed was divided by the observed (F.C.M.) yield. The distribution of the resulting ratios is shown in Fig. 6. The mean ratio is 1.0001, indicating a very good average agreement. The standard deviation of the ratios is .0194, and accordingly the computed yield equals the observed yield ± 1.3 percent of the observed yield. It appears, therefore, that the curves have been determined with a very fair degree of accuracy, as judged by the areas. (See also Table 17.)

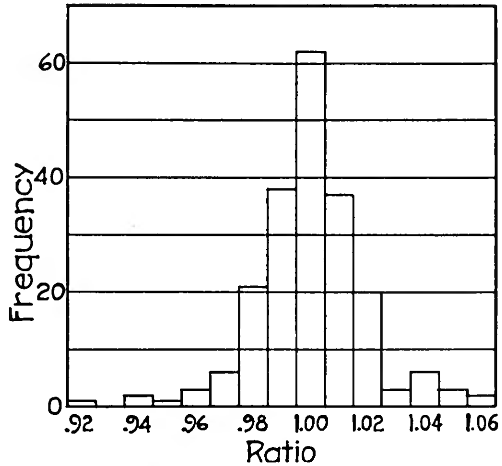


FIG. 6.—SHOWING THE RATIOS OF COMPUTED YIELDS TO ACTUAL YIELDS

The data give an indication of the accuracy of the estimation of the constants of the lactation curve to the observed values. See also Table 17.

The Holstein records studied consist, for each cow, of two 7-day records in the same lactation, the first of which was started not less than six days after calving and the second not less than eight months after calving. They afford thus only two observations for the determination of the lactation curve. In some cases more than one 7-day record eight months after calving is reported, and in such cases only the first in point of time is here considered.

The same curve as above described has been fitted algebraically to these two observations. Obviously this does not give as satisfactory a basis as the eleven observations of the Guernsey records, but it is all the published data afford. Certain features of the derivation of the lactation

curve from the Holstein records are pointed out in connection with Fig. 7.

The yields are considered, as in the Guernsey data, on an energy basis in terms of F.C.M. The constants of the lactation curve are computed thus: $k = \frac{\log_e y_1 - \log_e y_2}{t_2 - t_1}$, and $A = \log_e^{-1} (\log_e y_1 + kt_1)$, where y_1 = yield for the week of the first test in pounds F.C.M.; y_2 = yield for the week of the second test in pounds F.C.M.; t_1 = time in

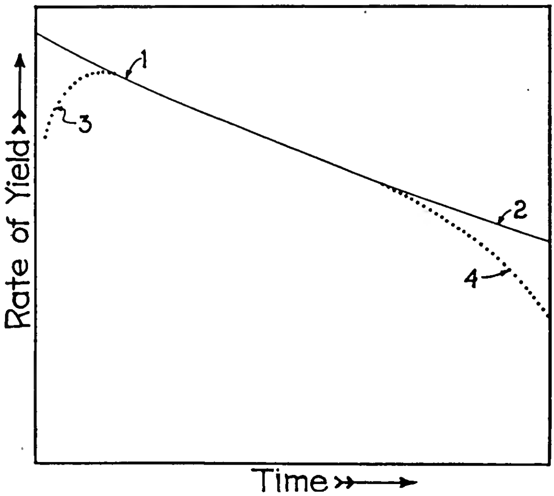


FIG. 7.—ILLUSTRATING CERTAIN CONDITIONS IN THE LACTATION CURVES DERIVED FROM HOLSTEIN RECORDS

The solid line represents the curve of equation (1). The dotted lines represent deviations from (1) associated with the preceding and concurrent pregnancies (cf. Fig. 24[6]). If the tests are conducted at 1 and 2 we may expect normal relations; if at 1 and 4, too high values for k and A; if at 3 and 2, too low values for k and A; if at 3 and 4, too low a value for A. The Holstein records give no information as to occurrence of conception. The influence of pregnancy illustrated at the right of the diagram does not become appreciable before the fifth or sixth month of gestation. The effect on the lactation curve has been avoided in the Guernsey records by dealing only with those records in which gestation was not far enough advanced to be a material factor.

months from calving to middle of first test; and t_2 = time in months from calving to middle of second test. As in the Guernsey data, k expresses the rate of decrease per month in the rate of yield; but the rate of yield is expressed in pounds F.C.M. *per week.* The value of A in the Holstein lactation curves has therefore to be divided by 7 in order to be directly comparable with A in the Guernsey lactation curves. A table of natural logarithms and a 20-inch slide rule have been used freely in the computations.

Records in which more than 139 days elapsed from calving to the start of the first test were excluded; also records were excluded in which less than 130 days elapsed between the start of the two tests. Nineteen

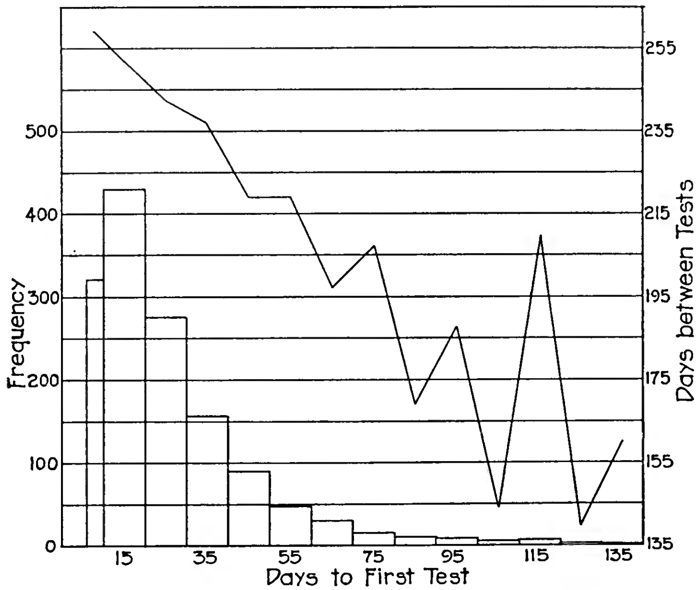


FIG. 8.—TIME OF CONDUCTING HOLSTEIN RECORDS

The columns show the distribution of the records with respect to the time from calving to start of the first test. The curve shows the corresponding mean time between the two tests. The correlation between the two times is $r = -.498 \pm .014$. The mean time after calving for the second test is about the same (around 270 days) regardless of the time of conducting the first test.

records, or 1.3 percent, were thus excluded. The distribution of the first tests with respect to the time after calving is given in Fig. 8.

Some further details of the methods used are given in connection with the presentation of the results.

RESULTS FROM GUERNSEY RECORDS

Perhaps one of the first points of interest in connection with persistency values is the form of the distribution curve of these values for the breed as a whole. It develops, however, that it is necessary to make certain corrections to the values, and the several relations involved in these corrections will be considered first.

Age and Persistency.—It is well known that milk yield is greatly affected by the age of the cow. It has been shown by Sanders[16] and by Gaines and Davidson[6] that persistency varies also with the age of the cow, older cows tending to be less persistent than younger cows. The mean persistency values for various age classes of the Guernsey records under study are given in Table 2 and shown graphically in Fig. 9.

TABLE 2.—VARIOUS AGE CLASSES AND CORRESPONDING MEAN VALUES FOR PERSISTENCY, THEORETICAL INITIAL RATE OF YIELD, AND YIELD FOR THE YEAR: GUERNSEY RECORDS

Age in years (class mid-points)	Number of records	Mean values		
		$k \times 10^3$ Persistency	*A* Initial rate of yield (pounds F.C.M. per day)	Yield for year (pounds F.C.M.)
1.25	2	35.0	24.0	7 000
1.75	46	36.7	29.8	8 674
2.25	410	32.2	33.1	10 020
2.75	202	33.6	35.2	10 559
3.25	137	44.0	39.4	11 202
3.75	130	46.0	39.7	11 092
4.25	117	51.2	43.3	11 756
4.75	70	59.3	45.1	11 657
5.25	81	56.7	45.9	12 043
5.75	51	47.2	42.9	11 833
6.25	62	62.4	46.8	11 871
6.75	32	60.6	44.8	11 688
7.5	76	53.7	46.8	12 500
8.5	44	55.7	47.5	12 454
9.5	38	62.1	46.1	11 974
10.5	16	66.9	45.3	11 188
11.5	10	62.0	43.6	11 100
12.5	4	52.5	47.5	12 750
13.5	3	35.0	34.0	10 500
14.5	2	75.0	49.0	11 000
15.5	0	....	....	......
16.5	1	65.0	40.0	9 500

It will be observed that the relation of the *k* values to age, while somewhat irregular, is certainly not linear. The Maine investigators have used an equation of the general form, $y = a + bx + cx^2 + d \log x$, to express the relation between age and yield, and the precedent of their usage is followed in choosing an equation to describe the age-persistency data. The equation is a purely empirical one, and no particular biological significance can be attached to any of its constants. As a mathematical description, however, it seems well adapted to the data.

The equation has been fitted to the age-persistency data by the "star-point method" of Smith.[17] This is essentially a "cut and try"

graphic method. The smooth curve thus derived has a close resemblance to the usual relation found between age and yield. Further information as to the relation between persistency and age is afforded by consideration of the several interrelations of the variables listed in Table 2.

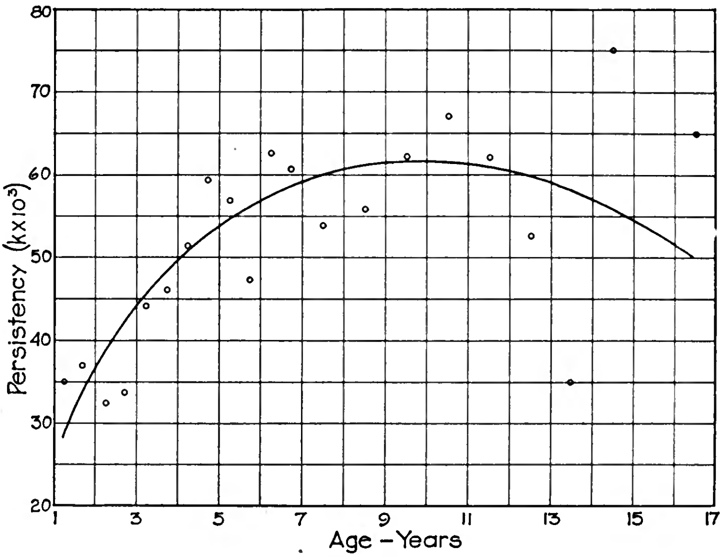


FIG. 9.—ILLUSTRATING THE CHANGE IN PERSISTENCY WITH ADVANCING AGE, GUERNSEY RECORDS

Equation of the curve: $y = 21.8 + 3.00x - .2206x^2 + 31.78 \log x$, x being the age in years. According to the equation y reaches a maximum at $x = 9.945 =$ 9 years, 11 months, 10 days, at which time $y = 61.5$. That is, the rate of decrease in the rate of yield per month at that age is 61.5 per mille per month.

In the equation $y = a + bx + cx^2 + d \log_{10}x$, the value of x at which y reaches either a maximum or a minimum is obtained by differentiating y with respect to x, setting the first derivative equal to zero, and solving for x. This gives $\frac{dy}{dx} = b + 2cx + d\left(\frac{1}{\log_e 10}\right)\left(\frac{1}{x}\right) = 0$, and $2cx^2 + bx + .4343d = 0$, from which we obtain

$$x = \frac{-b \pm (b^2 - 3.4744\,cd)^{1/2}}{4c}.$$

Age and Initial Rate of Yield.—The mean initial rates of yield for several age classes are given in Table 2 and shown graphically in Fig. 10. The smooth curve is of the same type as above and has been fitted by the method of moments as developed by Miner[14] with some final arbitrary adjustments in the constants. Apparently the relation between age and theoretical initial rate of yield is more regular than is the relation between age and persistency. A general parallelism between the two curves, Figs. 9 and 10, is evident.

Age and Yield.—The change in yield with age has been extensively studied. Since we are dealing here with a comparatively small population, the age-yield data afford something of a check on the representativeness of the selected records. The numerical data are given in Table 2, the graphic presentation in Fig. 11.

It will be noticed that all three of the age curves are more or less similar, indicating that there is a positive correlation between A and k of the lactation curve equation. The method of partial correlation would give an insight into the relation between any two of the variables

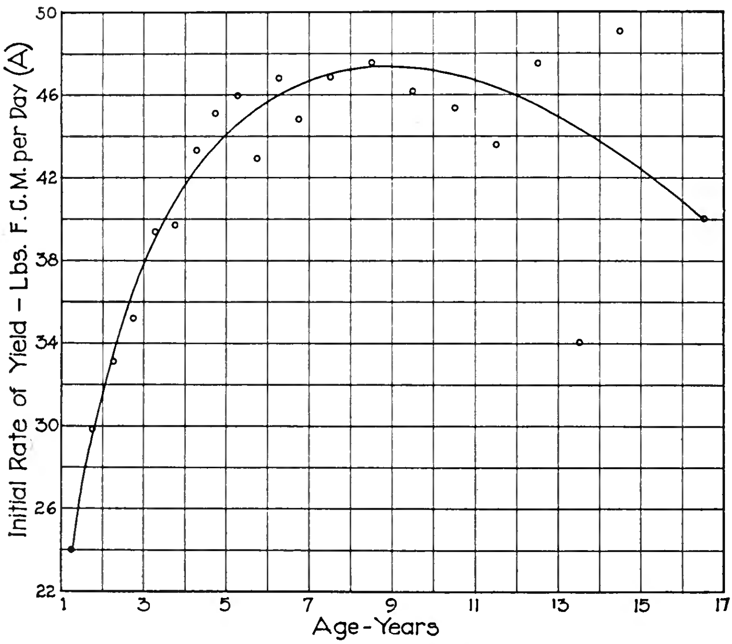


FIG. 10.—SHOWING THE CHANGE IN INITIAL RATE OF YIELD WITH ADVANCING AGE, GUERNSEY RECORDS

Equation of the curve: $y = 10.56 - .534x - .0132x^2 + 40.317 \log x$, where x is age in units of 6 months with origin at 1.5 months. This gives a maximum for y at $x = 17.554 = 8$ years, 10 months, 25 days of age. At this age the rate of yield is 47.3 pounds of F.C.M. per day by the equation.

with other variables constant, if the regressions were linear. Plainly the condition of linear regression is not satisfied over the entire age range. From the data as plotted it is evident that linear regression may be approximated by dealing with ages under five years. These ages include 1,114 records, or 72.6 percent of the total, and are used in the correlation treatment following.

Persistency, Initial Rate, Age, and Yield.—The means, standard deviations, and coefficients of variability of the variables mentioned are given in Table 3. All the possible simple and partial correlations are given in Table 4. The simple correlations have been derived by the usual method, using class intervals of 10 in $k \times 10^3$; 2, in A; six months, in age; and 1,000 pounds, in F.C.M. yield. The partial correlation coefficients have been derived by the general formulae,

$$r_{12\cdot3} = \frac{r_{12} - (r_{13} \times r_{23})}{\left[(1 - r_{13}^2)\ (1 - r_{23}^2)\right]^{1/2}} \text{ and } r_{12\cdot34} = \frac{r_{12\cdot3} - (r_{14\cdot3} \times r_{24\cdot3})}{\left[(1 - r_{14\cdot3}^2)\ (1 - r_{24\cdot3}^2)\right]^{1/2}}.$$

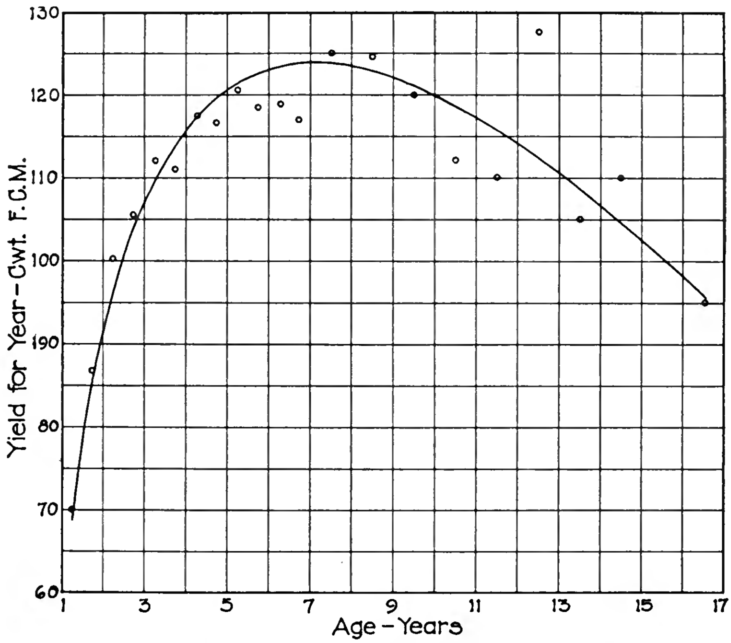


FIG. 11.—ILLUSTRATING THE CHANGE IN YIELD WITH ADVANCING AGE, GUERNSEY RECORDS

Equation of the curve: $y = 2393.8 - 434.14x + .2842x^2 + 13951.4 \log x$, x being the age in units of 6 months. Accordingly y reaches a maximum at $x = 14.129 = 7$ years, 24 days, at which age the yield for the year is 12,362 pounds F.C.M.

The simple correlations of Table 4 show that there is a significant correlation between any two of the variables. The closest relationship is evident between the initial rate of yield and the yield for the year. The initial rate of yield is also closely related to age and to persistency. The relation of persistency to age and to yield for the year is less close than the other relations noted.

TABLE 3.—STATISTICAL CONSTANTS OF THE VARIABLES INDICATED FOR AGES UNDER FIVE YEARS: GUERNSEY RECORDS

Constants	k × 10^3 Persistency	*A* Initial rate (pounds F.C.M. per day)	Age at calving (years)	Yield for year (pounds F.C.M.)
Mean..........	39.5 ± .6	36.70 ± .17	2.98 ± .02	10612 ± 41
Standard deviation.....	30.2 ± .4	8.63 ± .12	.85 ± .01	2307 ± 29
Coefficient of variability....	76.44 ± 1.09	23.51 ± .34	28.40 ± .25	21.74 ± .28

NOTE.—The probable errors of the coefficients of variability thruout this paper are computed by the approximate formula, $E_{cv} = \frac{100 E\sigma}{M}$.

TABLE 4.—SIMPLE AND PARTIAL COEFFICIENTS OF CORRELATION BETWEEN PERSISTENCY, INITIAL RATE OF YIELD, AGE, AND YIELD FOR THE YEAR: GUERNSEY RECORDS, UNDER FIVE YEARS

Variables correlated	Variables constant	Coefficients
Persistency and initial rate.......	....	.480 ± .016
Persistency and age.............	....	.268 ± .019
Persistency and yield............	....	−.267 ± .019
Initial rate and age..............	....	.490 ± .015
Initial rate and yield............	....	.691 ± .011
Age and yield..................	....	.308 ± .019
Persistency and initial rate.......	Age......................	.415 ± .017
	Yield.....................	.953 ± .002
	Age and yield...............	.935 ± .003
Persistency and age.............	Initial rate.................	.042 ± .020
	Yield.....................	.382 ± .017
	Initial rate and yield........	.009 ± .020
Persistency and yield............	Initial rate.................	−.943 ± .002
	Age......................	−.381 ± .017
	Initial rate and age..........	−.933 ± .003
Initial rate and age..............	Persistency.................	.428 ± .017
	Yield.....................	.403 ± .017
	Persistency and yield........	.141 ± .020
Initial rate and yield............	Persistency.................	.968 ± .001
	Age......................	.645 ± .012
	Persistency and age.........	.955 ± .002
Age and yield..................	Persistency.................	.409 ± .017
	Initial rate.................	−.048 ± .020
	Persistency and initial rate..	−.025 ± .020

The question arises, to what extent is persistency related to age with initial rate of yield constant, and to initial rate of yield with age constant? The partial correlation coefficients give an index to these relations. Persistency is closely related to initial rate of yield with age constant, the partial correlation being $.415 \pm .017$. On the other hand, with initial rate of yield constant, persistency appears to be independent of age, as shown by the coefficient $.042 \pm .020$. This relationship is significant in considering the proper correction of the persistency values.

Some of the partial correlations of Table 4 have no particular meaning except as a rough check on the computations. For example, if yield is constant, persistency and initial rate of yield bear to each other a definite relation and we should expect a correlation of 1. The correlation found is .953. Two reasons are apparent for the failure to realize the perfect correlation; first, the regressions are not exactly linear; and, second, the persistency and initial rate values of the lactation curves have not been perfectly adjusted to the observed yield. In connection with the latter reason see Fig. 6. The low partial correlation between persistency and age, with initial rate of yield and total yield constant, is to be expected since the constant factors automatically require that persistency also be constant, while age remains variable. Similar comment might be offered for others of the partial correlations, but in general it may be said that they show a fair degree of consistency in the data.

Correction Factors.—The evidence on the relations between persistency and age, and between persistency and initial rate of yield, show that a correction factor for persistency should be based on the initial rate of yield rather than on age. The correlations indicate quite clearly that such relation as exists between persistency and age is associated with the relation between age and initial rate of yield. The correlations, of course, say nothing as to whether persistency is due to initial rate, or whether initial rate is due to persistency. It seems reasonable to presume that persistency is distinctly affected by the initial rate of yield, and that for purposes of comparison correction of the persistency values should be made accordingly.

We shall return to the full number of records in deriving the persistency correction factors. The correlation surface is given in Table 5. The constants from the correlation surface are given in Table 6. The regression equation derived from the constants of Table 6 is $k \times 10_3 = 1.782\ A - 25.59$. The curve of this equation and the mean persistency values of the several initial rate classes are shown in Fig. 12.

The mean persistency values given in Fig. 12 show a quite regular agreement with the linear curve with a few exceptions at the extremes.

TABLE 5.—CORRELATION SURFACE FOR INITIAL RATE OF YIELD AND PERSISTENCY: GUERNSEY RECORDS

Initial rate of yield, pounds F.C.M. per day (A): class mid-points

Persistency, rate of decrease per mille per month ($k \times 10^3$): class mid-points

	18	20	22	24	26	28	30	32	34	36	38	40	42	44	46	48	50	52	54	56	58	60	62	64	66	68	70	72	74	76	78	Total
−35	..	..	2	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	2
−25	..	1	1	..	..	..	2	..	1	1	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	6
−15	2	1	..	4	2	2	2	1	3	1	..	1	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	19
− 5	..	1	4	3	7	6	5	9	4	5	6	2	4	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	56
5	..	3	2	7	3	5	7	6	11	16	10	3	5	2	2	2	2	..	..	..	1	..	..	..	..	..	..	..	..	..	..	87
15	..	5	6	10	10	11	22	15	9	22	18	13	9	8	2	2	3	1	..	..	1	1	..	..	..	..	..	..	..	..	..	168
25	..	..	1	6	14	23	21	18	27	13	21	22	19	9	6	6	3	6	1	2	1	2	..	..	1	..	1	..	..	..	..	223
35	..	1	3	5	8	11	12	23	20	28	12	19	15	16	12	9	6	6	2	..	2	3	1	..	1	..	..	..	..	..	..	215
45	..	1	1	1	4	7	11	11	13	21	16	16	20	12	13	8	7	3	1	2	2	1	2	..	..	..	..	..	..	..	..	173
55	..	..	..	..	1	7	16	11	18	11	16	11	16	12	11	7	4	9	3	3	..	2	1	..	..	..	..	..	1	..	..	160
65	..	..	..	1	4	1	4	7	14	5	8	15	16	8	5	10	5	7	4	7	2	5	1	..	..	..	..	..	..	..	..	129
75	..	..	..	..	..	4	2	6	6	7	10	12	12	7	12	5	4	2	2	2	6	2	..	..	..	..	..	..	..	..	..	101
85	..	..	..	..	1	..	1	5	2	5	7	4	3	6	2	10	8	5	7	3	5	1	1	1	1	..	..	..	..	..	..	78
95	..	..	..	..	..	..	..	4	1	2	1	3	3	4	..	4	6	4	1	2	3	2	2	1	.	..	.	..	2	..	..	45
105	..	..	..	..	..	..	..	3	..	1	2	1	2	4	2	1	1	3	2	2	..	1	..	1	..	..	..	..	..	..	..	26
115	..	..	..	..	..	..	..	..	..	1	..	2	3	..	1	..	2	4	..	1	..	1	..	..	1	..	..	..	..	..	..	16
125	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	2	1	1	..	..	..	1	..	..	..	..	1	..	..	..	6
135	..	..	..	..	..	..	..	..	..	..	..	..	..	..	1	1	..	2	..	1	..	1	..	1	..	..	..	..	..	..	..	7
145	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	1	..	1	..	..	1	..	1	..	..	..	..	..	..	4
155	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	1	..	..	..	1	2	..	..	..	..	..	4
165	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	1	..	..	..	..	..	..	1	2
175	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	1	1
185	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	1	..	..	..	..	..	1
195	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	1	..	..	..	..	1	..	..	..	..	1	..	..	..	..	3
205	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	1	..	1
215	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	1	..	..	..	..	..	..	..	..	..	1
Total	2	13	20	37	54	77	105	119	129	139	127	124	127	88	69	65	54	54	24	26	24	24	10	5	6	3	2	1	3	1	2	1534

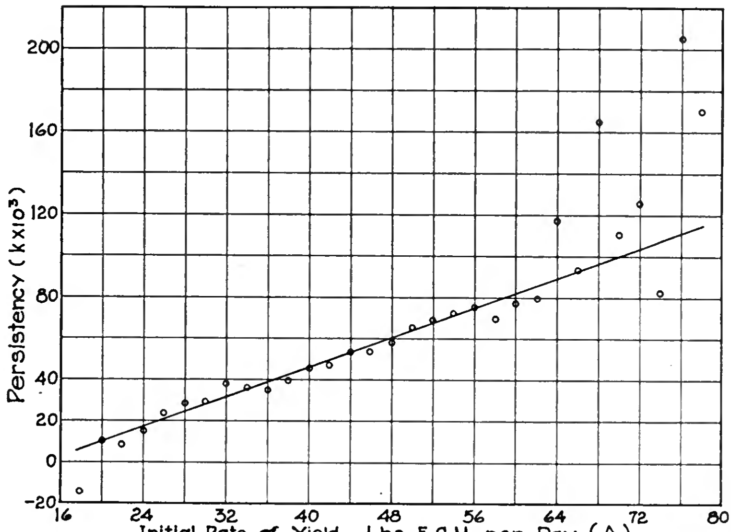


FIG. 12.—SHOWING REGRESSION OF PERSISTENCY VALUES ON INITIAL RATE OF YIELD, GUERNSEY RECORDS

Equation of the curve: $y = 1.782x - 25.59$.

The frequencies here are small, which may account for the variation in the mean values. It is possible that the entrance requirements of the advanced registry may have some effect at the lowest initial rate values. Thus a cow that starts with a rate of 18 pounds F.C.M. per day would need to have a persistency of about $k = 0$ in order to satisfy the mini-

TABLE 6.—STATISTICAL CONSTANTS FOR INITIAL RATE OF YIELD AND PERSISTENCY: GUERNSEY RECORDS

Constants	$k \times 10^3$ Persistency	*A* Initial rate of yield (pounds F.C.M. per day)
Mean	$44.25 \pm .55$	$39.19 \pm .17$
Standard deviation	$32.19 \pm .39$	$9.67 \pm .12$
Coefficient of variability	$72.75 \pm .89$	$24.67 \pm .30$
Coefficient of correlation	$.535 \pm .012$	

mum requirement of 250.5 pounds of fat at two years of age or under. Judging by the data of Fig. 12, no selective action is apparent except possibly in the case of the two cows of the 18-pound class.

The correction factors for the persistency values are derived from the above equation $k \times 10^3 = 1.782A - 25.59$, to correct to the mean

value of A, 39.19. Correction to this value of A permits a minimum correction to a maximum number of the records. The correction formula is $k_c = k_o + 69.84 - 1.782\ A$, where k_o is the observed $k \times 10^3$, and k_c is the corresponding corrected value.

The age-correction factors to be applied to the yield for the year are derived from the equation given in Fig. 11 by the direct-ratio method used by Gowen[8] (chap. 4). The age of maximum yield is used as the base, or standard age.

The age correction factors for the initial rate of yield are derived from the equation of Fig. 10 on the same basis as above.

TABLE 7.—MONTH OF CALVING AND MEAN VALUES FOR PERSISTENCY, INITIAL RATE OF YIELD, YIELD FOR YEAR, AND FAT PERCENTAGE: GUERNSEY RECORDS

Month of calving	Number of records	$k \times 10^3$ Persistency corrected for A	A Initial rate of yield corrected for age (pounds F.C.M. per day)	Yield for year corrected for age (pounds F.C.M.)	Fat percentage for year
January....	95	48.9	49.2	12 700	5.05
February...	128	49.9	49.0	12 297	5.15
March......	166	50.0	52.2	12 855	5.05
April.......	158	50.8	48.4	12 411	5.06
May........	126	49.8	45.6	12 040	5.12
June........	96	37.3	44.0	12 646	5.01
July........	88	36.7	41.2	12 023	5.02
August......	123	37.0	42.8	12 476	5.03
September..	134	37.2	45.2	12 731	5.07
October.....	165	40.4	47.6	12 936	4.98
November..	125	42.1	48.4	12 732	5.09
December...	130	52.5	50.4	12 508	5.00

It will be understood that the correction of milk yield for fat content (page 359) is really not a correction in the same sense as the age correction. It is rather a method of estimating and expressing the energy value of the milk.

Influence of Season.—The season of the year at which the cow calves may have a considerable influence on her production under ordinary conditions, largely because of the feed supplied from pastures. Sanders[16] has shown that the season of calving affects persistency under the usual practice of management. This seasonal effect is much less important in cows on official test because of the better and more uniform care and feed which they receive.

The data are tabulated in Table 7 according to the month in which the cow calved, and are also given in Fig. 13. It is apparent that the

total yield for the year is but little affected by the month of calving. The lowest yields occur with calving in May and July. The effect of the time of calving on initial rate of yield is somewhat more pronounced and regular, tending toward a low point for July calvers. The persistency values also show a pronounced drop in June, which continues for the summer months. Apparently cows calving in the summer months hold up somewhat better than cows calving in the winter months when the initial rate of yield is allowed for. No direct corrections are made for the effect of month of calving on persistency or initial rate of yield.

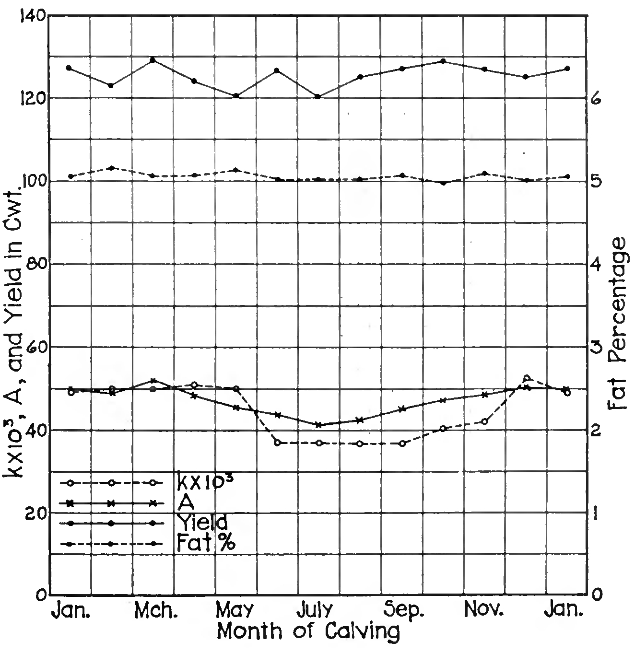


FIG. 13.—EFFECT OF THE SEASON OF FRESHENING, GUERNSEY RECORDS

The initial rate of yield (A) in pounds of F.C.M. per day is corrected for age. Persistency, ($k \times 10^3$) rate of decrease per mille per month, is corrected for A. The yield for the year in cwt. F.C.M. (Yield) is corrected for age. The fat percentage for the year (Fat %) is taken directly from the records.

Variability in Persistency.—The distribution of the observed persistency values has been given in Table 5, and the constants derived from this array have been given in Table 6. The mean k value is .04425[a];

[a]The same records under consideration here have been previously[6] dealt with in groups classified on the basis of the length of the gestation included in the record period. It may be noted that the k values previously determined from the group records varied from .03714 to .04361, and are thus lower than the mean here reported. The group records include contemporaneous reëntries. How far the reëntry records

standard deviation, .03219; and coefficient of variability, 72.75.

Correction of these values on the basis presented above leads to the distribution given in Table 8. The two distributions are shown graphically on a percentage basis in Fig. 14. The corrected persistency

TABLE 8.—VARIABILITY IN CORRECTED PERSISTENCY VALUES AND ACCOMPANYING VARIABILITY IN CALCULATED YIELD: GUERNSEY RECORDS

(The persistency values have been corrected for initial rate of yield to A = 39.19)

Frequency	$k \times 10^3$ Persistency corrected for A (class mid-points)	Yield for year A = 39.19 (pounds F.C.M.)
1	− 35	17 821
3	− 25	16 723
13	− 15	15 712
34	− 5	14 779
79	5	13 918
143	15	13 123
196	25	12 389
226	35	11 710
241	45	11 080
181	55	10 498
161	65	9 957
109	75	9 455
56	85	8 989
43	95	8 556
25	105	8 153
10	115	7 777
8	125	7 427
0	135	......
2	145	6 795
0	155	......
1	165	6 242
2	175	5 992
Mean	44.81	11 260
Standard deviation	27.32	1 636
Coefficient of variability	60.97	14.53

may be responsible for the lower k values of the group records in which they appear has not been determined. The primary purpose of this note, however, is to point out that it is not quite proper to assume that the average of the k's is the same as the k obtained from the average of the monthly data. As an extreme example, if we have two records with the same initial rate but in one record k = 0 and in the other k =.2, the mean of the k's is .1; but if the two records are thrown together, the value of k determined from the group record would be, not .1, but about .06. Furthermore, this group lactation curve would not conform well to the curve of the exponential equation, having a considerably greater curvature than the equation provides. Now the observed group lactation curves previously presented show precisely this feature in a very small degree. But it would be straining the point perhaps to explain the deviations of the observed curves on the basis of their being made up of records which individually correspond to the equation and therefore collectively differ from the equation in the direction found.

values give a somewhat more regular distribution curve than the raw values. The correction has, of course, caused a change of places among the individuals. The mean has been slightly increased and the standard deviation considerably lowered. The coefficient of variability is still very high as compared with the same constant for other characters. It is doubtful if this constant has its usual significance in the present case.

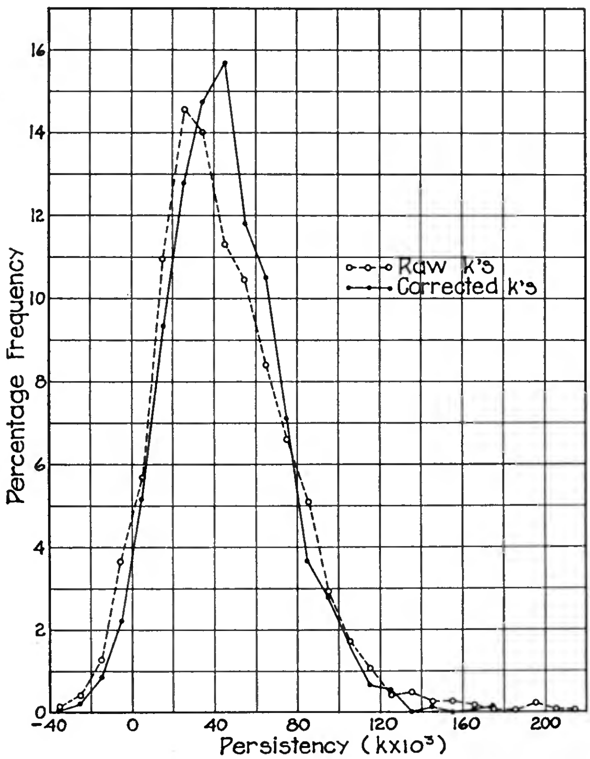


FIG. 14.—PERCENTAGE FREQUENCY DISTRIBUTIONS FOR PERSISTENCY OF LACTATION, GUERNSEY RECORDS

The corrected *k's* have been corrected for initial rate of yield as explained in text.

Where the variability in persistency is based on a constant value of initial rate of yield, the yields for the year may be computed. The last column of Table 8 gives these computed yields on the basis of the mean initial rate. These values give 14.53 as the coefficient of variability. The persistency distribution of the two initial-rate arrays of Table 5 at $A = 38$ and 40, by the same treatment give a coefficient of variability

in yield of 14.00. It appears, therefore, that the natural variability in persistency of Guernsey cows under the conditions and prescriptions of official test is responsible for a standard deviation in yearly yield equal to about 14.5 percent of the mean yield.

Variability in Initial Rate of Yield.—The distribution of the initial-rate-of-yield values has been given in Table 5. The coefficient of

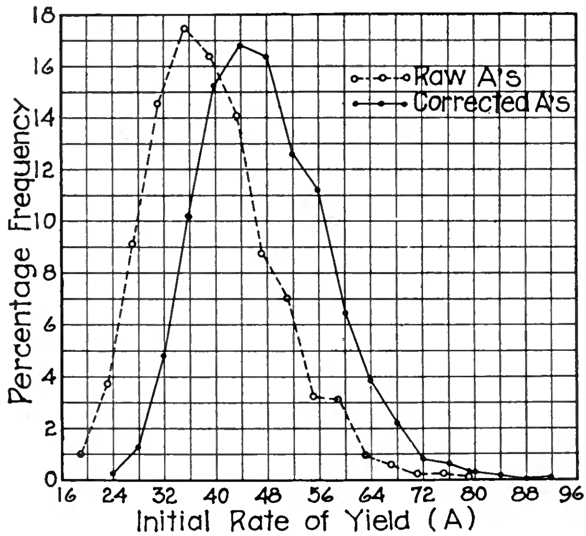


FIG. 15.—PERCENTAGE FREQUENCY DISTRIBUTIONS FOR INITIAL RATE OF YIELD, GUERNSEY RECORDS

The corrected *A's* have been corrected for age of the cow at calving.

TABLE 9.—VARIABILITY IN INITIAL RATE OF YIELD CORRECTED FOR AGE: GUERNSEY RECORDS

(Mean = 47.22; standard deviation = 9.62; coefficient of variability = 20.37)

A[a]......	24	28	32	36	40	44	48	52	56	60	64	68	72	76	80	84	88	92
f[b]......	4	19	74	156	233	257	251	193	141	99	59	19	12	10	4	2	0	1

[a]Class mid-points of initial rate of yield in pounds of F.C.M. per day.
[b]Frequency.

variability (Table 6) is 24.67. Corrected for age, the values are given in Table 9. The distributions for the raw and corrected values are given graphically on a percentage basis in Fig. 15.

The mean initial rate is of course increased by the age correction since the age of maximum initial rate has been used as a base. The mean of the age-corrected data is 47.22; the standard deviation is 9.62; the coefficient of variability, 20.37.

This coefficient is quite directly comparable with the same measure of variability in yearly yield, since if persistency is constant the yield for the year is proportional to the initial rate and the coefficients of variability would therefore be the same. The constants of the age- and fat-corrected observed milk yields are: mean, 12,553; standard deviation, 2,503; coefficient of variability, 19.94.

TABLE 10.—AVERAGE DAILY YIELDS BY MONTHS FOR SPECIAL GROUP OF COWS: GUERNSEY RECORDS

(Each record shows an *increasing* rate of yield with advance in lactation.)

Months after calving	Number of records	Milk per day (pounds)	Fat per day (pounds)	Fat percentage	F.C.M. per day (pounds)
1..........	72	30.0	1.33	4.44	32.0
2..........	83	28.6	1.31	4.59	31.2
3..........	83	27.5	1.31	4.77	30.6
4..........	83	26.9	1.32	4.92	30.6
5..........	83	26.7	1.36	5.09	31.1
6..........	83	26.7	1.38	5.17	31.4
7..........	83	27.1	1.42	5.22	32.1
8..........	83	27.1	1.43	5.27	32.3
9..........	83	27.6	1.46	5.29	32.9
10..........	83	27.5	1.46	5.29	32.8
11..........	83	26.8	1.44	5.37	32.3

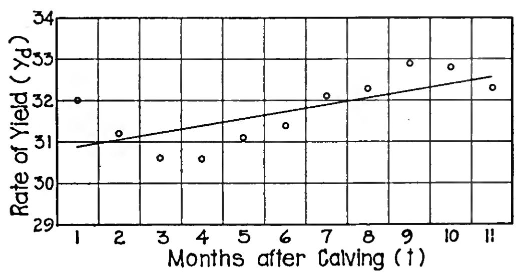


FIG. 16.—SHOWING RATE OF YIELD FOR THE SPECIAL GROUP OF COWS, GUERNSEY RECORDS

Each of the 83 records shows an increasing rate of yield with advance in lactation. The equation of the curve fitted by the method of least squares to the data of Table 10 is $y_d = 30.725e^{.005439t}$.

Increasing Rate of Yield with Advance in Lactation.—Advance in lactation usually implies a decrease in the rate of energy yield. A number of the records studied show an increase in the rate of energy yield with advance in lactation. In Table 5 there are 83 such records, constituting 5.41 percent of the total. These records are of special interest because of their unusual, if not abnormal, nature. The results from them,

treated as a group, are given in Table 10 and Fig. 16. It is evident from Fig. 16 that these records give an average lactation curve which is not in very good conformity with the normal lactation curve. The first three observations show a normally decreasing rate of yield, after which there is a distinct trend toward an increasing rate, and finally, in the eleventh observation, there is evidence that a decreasing rate again has set in. It will be observed further that the initial rate of yield is much lower than the mean of all the records. This relation implies the presence of a disproportionate number of young cows.

It is evident that the equation used does not afford a complete description of the observed lactation curve for the records of Fig. 10 as a group. On the other hand, any simple expression of the observed lactation curve must recognize a general tendency for the rate of yield to increase with advance in lactation, and this requirement the equation answers satisfactorily for the limited time range.

Rate of Yield and Yield for the Year.—It is intended under this head to consider the relation between the yearly yield and the rate of yield at various points in the lactation curve, for various age classes, and for all ages together. We have already seen (Table 4) that there is a marked correlation (r = .691) between initial rate and yearly yield, all records taken together. The question is, how does this correlation between rate and yearly yield change as we consider the rate of yield at later times in the lactation; first, with age relatively constant; and second, without any age selection. Evidently a series of correlations may be used to indicate the common point in the lactation curves at which the rate of yield is most closely related to the year's yield. Such a point of closest relationship should be, theoretically, the ideal time to determine, by a short-time test, the probable year's production. Any differences due to, or associated with the age of the cow should be brought to light by such a series of correlations.

The rates of yield, determined by the constants of the equations, have been computed for each lactation curve at ten additional points; namely, 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10 months after calving. The rate at ten months after calving was first computed algebraically by the use of a table of exponentials. The rates at the intermediate points were computed graphically by the use of an aritho-log chart improvised for the purpose. The 0 and 10 logarithmic ordinates consisted of the scales of a 20-inch slide rule. Connecting these two logarithmic ordinates, abscissas were drawn for the 2-pound class intervals used. Nine intermediate ordinates were drawn to give equal spacing. A taut thread was properly adjusted to the end ordinates according to the two corresponding previously-determined values for a particular record. The points

TABLE 11.—CORRELATION BETWEEN THEORETICAL RATE OF YIELD AT VARIOUS TIMES AFTER CALVING AND OBSERVED YIELD FOR THE YEAR: GUERNSEY RECORDS

Months after calving	Age at calving (years)							
	1 to 2	2 to 3	3 to 4	4 to 5	5 to 6	6 to 7	7 to 8	8 to 17
	Coefficient of correlation between theoretical rate of yield and observed yield for year							
0	.629 ± .059	.728 ± .013	.577 ± .028	.588 ± .032	.555 ± .041	.539 ± .049	.485 ± .059	.545 ± .044
1	.715 ± .048	.807 ± .010	.693 ± .021	.712 ± .024	.677 ± .032	.657 ± .040	.631 ± .047	.697 ± .032
2	.820 ± .032	.872 ± .007	.803 ± .015	.821 ± .016	.790 ± .022	.789 ± .026	.757 ± .033	.814 ± .021
3	.868 ± .024	.925 ± .004	.893 ± .008	.905 ± .009	.887 ± .013	.887 ± .015	.862 ± .020	.899 ± .012
4	.927 ± .014	.959 ± .002	.948 ± .004	.955 ± .004	.944 ± .006	.948 ± .007	.936 ± .010	.952 ± .006
5	.973 ± .005	.977 ± .001	.977 ± .002	.980 ± .002	.973 ± .003	.974 ± .004	.970 ± .005	.968 ± .004
6	.974 ± .005	.980 ± .001	.979 ± .002	.982 ± .002	.976 ± .003	.971 ± .004	.979 ± .003	.969 ± .004
7	.971 ± .006	.973 ± .002	.967 ± .003	.969 ± .003	.962 ± .004	.952 ± .007	.971 ± .004	.961 ± .005
8	.945 ± .010	.962 ± .002	.943 ± .005	.948 ± .005	.939 ± .007	.923 ± .010	.959 ± .006	.937 ± .008
9	.931 ± .013	.943 ± .003	.913 ± .007	.927 ± .007	.909 ± .010	.889 ± .015	.940 ± .009	.915 ± .010
10	.887 ± .024	.920 ± .004	.884 ± .009	.903 ± .009	.873 ± .014	.865 ± .018	.920 ± .012	.894 ± .012

where the thread cut the intermediate ordinates served to show the corresponding intermediate class values for the lactation curve.

The values thus obtained were correlated with the observed yield for the year, both for all the records together and for various age classes. The correlation coefficients for the various age classes are given in

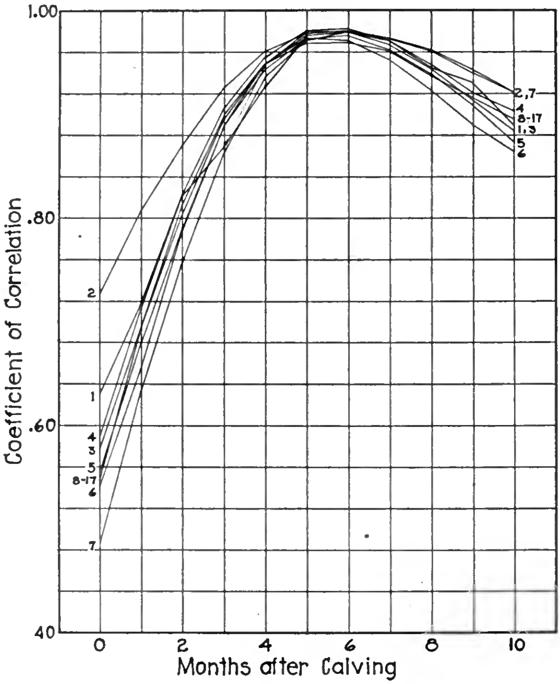


FIG. 17.—CORRELATION BETWEEN THE THEORETICAL RATE OF YIELD AND THE OBSERVED YIELD FOR THE YEAR, AGE-CLASSIFIED, GUERNSEY RECORDS

This is a graphic representation of the data of Table 11. The figures at left and right of the curves indicate the age classes. Note the rapid rise of the curves to 6 months and their convergence between 5 and 6 months after calving. Theoretically a short-time test will afford the best index of the year's production if conducted during the fifth month of lactation. At this point in the lactation curve there is no marked or significant difference between the age groups. At earlier points in the lactation curve the rate of yield for two-year-old cows seems to be distinctly more closely related to the year's production than is the case for any other age.

Table 11. It will be observed from the data of Table 11 that there is considerable variation in the coefficients at the initial point of the lactation curve. The highest correlation, .728, is found for two-year-old cows; and the lowest, .485, for seven-year-old cows. These relative positions are maintained also at 1, 2, 3, and 4 months after calving. It would seem, therefore, that any short-time test conducted shortly after

calving should afford a better index of the year's production in the case of two-year-old, or young cows, than in the case of seven-year-old, or old cows.

A more remarkable fact brought out by Table 11 is the very material increase in the coefficients at later points on the lactation curve up to

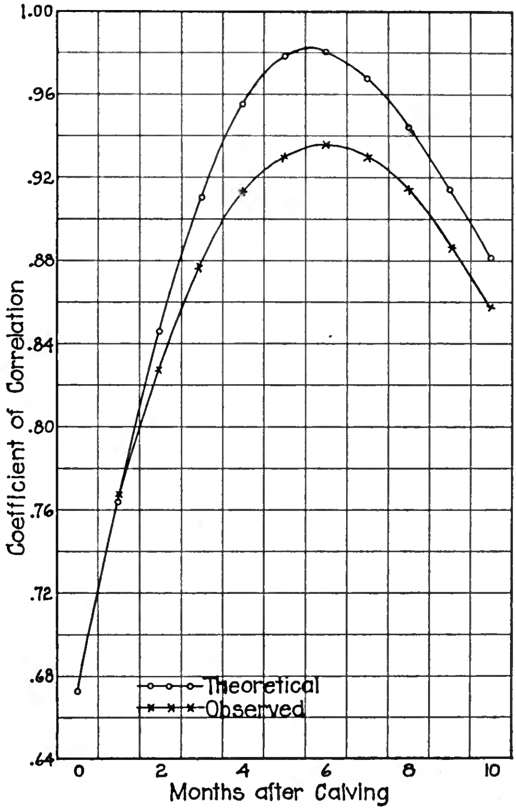


FIG. 18.—SHOWING THE CORRELATION BETWEEN RATE OF YIELD AT VARIOUS TIMES AFTER CALVING AND YIELD FOR THE YEAR, ALL AGES, GUERNSEY RECORDS

The theoretical curve is based on the fitted lactation curves. The observed curve is based on the actual lactation curves. Data from Table 17. The smooth curves given in the graph are merely free-hand sketches.

six months after calving. At this point on the lactation curve the two-year-old group and the seven-year-old group have reached practically the same high value, .980. All of the other age groups show the highest correlation at six months, except the six-year-old group, where the highest coefficient is found at five months. The coefficients decrease in value after six months. These relations for the several age groups are shown graphically in Fig. 17.

Similar correlations have been computed for all ages lumped together. They fall naturally in between the extremes given in Table 11 and Fig. 17. The correlation surface for the theoretical initial rate of yield and yield for the year is given in Table 12. Similar surfaces at three months, six months, and ten months after calving are given in Tables 13, 14, and 15 respectively.

On account of the practical importance of the changes in the closeness of the relation between the rate of yield and yearly yield with

TABLE 12.—CORRELATION BETWEEN THEORETICAL INITIAL RATE OF YIELD AND TOTAL YIELD FOR THE YEAR: GUERNSEY RECORDS

Yield for year, hundredweight F.C.M.: class mid-points

Theoretical initial rate of yield, pounds F.C.M. per day: class mid-points	55	65	75	85	95	105	115	125	135	145	155	165	175	185	195	205	215	Total
18	..	..	2	..	..	..	..	..	..	..	..	..	..	..	..	..	..	2
20	2	7	2	2	..	..	..	..	..	..	..	..	..	..	..	..	..	13
22	1	4	7	5	1	1	1	..	..	..	..	..	..	..	..	..	..	20
24	1	..	15	17	4	..	..	..	..	..	..	..	..	..	..	..	..	37
26	1	4	17	20	6	6	..	..	..	..	..	..	..	..	..	..	..	54
28	..	4	14	31	20	6	2	..	..	..	..	..	..	..	..	..	..	77
30	..	1	16	25	38	16	7	2	..	..	..	..	..	..	..	..	..	105
32	..	5	16	20	38	23	10	7	..	..	..	..	..	..	..	..	..	119
34	..	..	3	32	24	36	20	8	5	1	..	..	..	..	..	..	..	129
36	..	..	6	13	25	34	29	22	9	1	..	..	..	..	..	..	..	139
38	..	..	2	11	21	28	20	28	14	3	..	..	..	..	..	..	..	127
40	..	..	1	6	19	21	31	28	14	2	2	..	..	..	..	..	..	124
42	..	..	2	6	11	26	27	23	21	6	4	1	..	..	..	..	..	127
44	..	..	..	2	12	10	19	16	18	5	6	..	..	..	..	..	..	88
46	..	..	..	2	2	8	13	16	18	6	3	1	..	..	..	..	..	69
48	..	..	..	1	2	13	11	12	8	12	2	2	2	..	..	..	..	65
50	..	..	1	..	4	6	11	7	6	9	5	2	3	..	..	..	..	54
52	..	..	..	2	2	11	4	7	10	5	8	4	1	..	..	..	..	54
54	..	..	..	..	1	2	4	5	5	4	2	..	1	..	..	..	..	24
56	..	..	..	..	2	..	5	4	5	3	5	..	2	..	..	..	..	26
58	..	..	..	..	1	..	1	5	7	3	..	4	..	1	2	..	..	24
60	..	..	1	..	1	1	1	3	1	4	4	2	3	2	1	..	..	24
62	..	..	..	..	..	1	1	1	1	1	2	..	2	1	..	..	..	10
64	..	..	..	..	1	..	1	..	2	1	..	..	..	..	..	..	..	5
66	..	..	..	..	..	2	..	1	..	1	..	..	..	..	2	..	..	6
68	..	..	..	..	..	2	1	..	..	..	..	..	..	..	..	..	..	3
70	..	..	..	1	..	..	..	..	..	..	..	..	..	..	..	..	1	2
72	..	..	..	..	..	..	..	..	1	..	..	..	..	..	..	..	..	1
74	..	..	..	..	..	..	..	..	..	1	..	1	..	..	1	..	..	3
76	..	..	..	..	1	..	..	..	..	..	..	..	..	..	..	..	..	1
78	..	..	..	..	..	1	1	..	..	..	..	..	..	..	..	..	..	2
Total	5	25	105	196	236	254	220	195	145	68	43	17	14	4	6	0	1	1534

advance in lactation, a similar comparison has been made between the observed monthly yields and yield for the year. One correlation surface at six months after calving is given in Table 16; that is, the surface is based on the month the mid-point of which is closest to six-months after calving. The constants derived from Tables 12 to 16, and from similar tables for other points in the lactation curves, are given in Table 17.

Table 17 shows also the highest correlation between rate of yield and yearly yield at six months after calving. For the theoretical or smoothed lactation curve, the correlation at six months after calving

is .980. The corresponding correlation for the actual yield of the sixth month and the yearly yield is .935. Both comparisons show the maximum correlation at six months after calving. The actual yield of the sixth

TABLE 13.—CORRELATION BETWEEN THEORETICAL RATE OF YIELD THREE MONTHS AFTER CALVING AND TOTAL YIELD FOR THE YEAR: GUERNSEY RECORDS

Yield for year, hundredweight F.C.M.: class mid-points

Theoretical rate of yield three months after calving, pounds F.C.M. per day: class mid-points	55	65	75	85	95	105	115	125	135	145	155	165	175	185	195	205	215	Total
18	4	2	2	..	..	..	..	..	..	..	..	..	..	..	..	..	..	8
20	1	9	3	1	..	..	..	..	..	..	..	..	..	..	..	..	..	14
22	..	9	25	7	..	..	..	..	..	..	..	..	..	..	..	..	..	41
24	..	5	41	38	5	1	..	..	..	..	..	..	..	..	..	..	..	90
26	..	..	23	62	26	3	1	..	..	..	..	..	..	..	..	..	..	115
28	..	..	6	55	70	16	1	..	..	..	..	..	..	..	..	..	..	148
30	..	..	4	22	60	53	10	..	..	..	..	..	..	..	..	..	..	149
32	..	..	1	8	42	72	30	9	..	..	..	..	..	..	..	..	..	162
34	..	..	..	2	21	51	53	20	1	..	..	..	..	..	..	..	..	148
36	..	..	..	..	8	26	63	54	13	1	..	..	..	..	..	..	..	165
38	..	..	..	1	3	21	36	53	34	5	..	..	..	..	..	..	..	153
40	..	..	..	..	..	8	15	32	45	7	1	..	..	..	..	..	..	108
42	..	..	..	..	1	2	7	13	20	13	8	..	..	..	..	..	..	64
44	..	..	..	..	..	..	2	12	15	19	6	1	..	..	..	..	..	55
46	..	..	..	..	..	..	2	2	13	12	11	2	1	..	..	..	..	43
48	..	..	..	..	..	1	..	..	2	7	11	6	3	..	..	..	..	30
50	..	..	..	..	..	..	..	..	2	2	4	2	4	..	..	..	..	14
52	..	..	..	..	..	..	..	..	..	1	2	5	1	..	..	..	..	9
54	..	..	..	..	..	..	..	..	..	..	..	..	5	1	..	..	..	6
56	..	..	..	..	..	..	..	..	..	1	..	1	..	3	2	..	..	7
58	..	..	..	..	..	..	..	..	..	..	..	..	..	..	2	..	..	2
60	..	..	..	..	..	..	..	..	..	..	..	..	..	..	1	..	..	1
62	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	0
64	..	..	..	..	..	..	..	..	..	..	..	..	..	..	1	..	1	2
Total	5	25	105	196	236	254	220	195	145	68	43	17	14	4	6	0	1	1534

TABLE 14.—CORRELATION BETWEEN THEORETICAL RATE OF YIELD SIX MONTHS AFTER CALVING AND TOTAL YIELD FOR THE YEAR: GUERNSEY RECORDS

Yield for year, hundredweight F.C.M.: class mid-points

Theoretical rate of yield six months after calving, pounds F.C.M. per day: class mid-points	55	65	75	85	95	105	115	125	135	145	155	165	175	185	195	205	215	Total
16	5	3	1	..	1	..	..	..	..	..	..	..	..	..	..	..	..	10
18	..	21	9	..	..	..	..	..	..	..	..	..	..	..	..	..	..	30
20	..	1	54	3	..	..	..	..	..	..	..	..	..	..	..	..	..	58
22	..	..	39	86	4	..	..	..	..	..	..	..	..	..	..	..	..	129
24	..	..	2	103	49	1	..	..	..	..	..	..	..	..	..	..	..	155
26	..	..	..	4	155	27	1	..	..	..	..	..	..	..	..	..	..	187
28	..	..	..	..	26	149	9	..	..	..	..	..	..	..	..	..	..	184
30	..	..	..	..	1	75	96	..	..	..	..	..	..	..	..	..	..	172
32	..	..	..	..	..	1	105	31	..	1	..	..	..	..	..	..	..	138
34	..	..	..	..	..	1	9	134	8	..	..	..	..	..	..	..	..	152
36	..	..	..	..	..	..	..	29	77	1	..	..	..	..	..	..	..	107
38	..	..	..	..	..	..	..	1	55	30	..	..	..	..	..	..	..	86
40	..	..	..	..	..	..	..	..	5	32	9	..	..	..	..	..	..	46
42	..	..	..	..	..	..	..	..	..	4	23	1	..	..	..	..	..	28
44	..	..	..	..	..	..	..	..	..	..	11	9	..	..	..	..	..	20
46	..	..	..	..	..	..	..	..	..	..	..	6	6	..	..	..	..	12
48	..	..	..	..	..	..	..	..	..	..	..	1	7	..	..	..	..	8
50	..	..	..	..	..	..	..	..	..	..	..	..	1	2	..	..	..	3
52	..	..	..	..	..	..	..	..	..	..	..	..	..	2	1	..	..	3
54	..	..	..	..	..	..	..	..	..	..	..	..	..	..	2	..	..	2
56	..	..	..	..	..	..	..	..	..	..	..	..	..	..	3	..	..	3
58	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	0
60	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	1	1
Total	5	25	105	196	236	254	220	195	145	68	43	17	14	4	6	0	1	1534

TABLE 15.—CORRELATION BETWEEN THEORETICAL RATE OF YIELD TEN MONTHS AFTER CALVING AND TOTAL YIELD FOR THE YEAR: GUERNSEY RECORDS

Yield for year, hundredweight F.C.M.: class mid-points

Theoretical rate of yield ten months after calving, pounds F.C.M. per day: class mid-points	55	65	75	85	95	105	115	125	135	145	155	165	175	185	195	205	215	Total
6	..	..	..	..	1	..	..	..	..	..	..	..	..	..	..	..	..	1
8	..	..	2	..	..	..	..	..	..	..	..	..	..	..	..	..	..	2
10	1	2	..	1	1	..	..	..	..	..	..	..	..	..	..	..	..	5
12	1	3	5	4	3	1	..	..	..	..	..	..	..	..	..	..	..	17
14	3	9	18	8	3	3	1	..	..	..	..	..	..	..	..	..	..	45
16	..	3	28	26	13	7	1	..	..	..	..	..	..	..	..	..	..	78
18	..	7	34	52	19	12	4	..	..	..	..	..	..	..	..	..	..	128
20	..	1	16	46	49	32	7	2	..	..	..	..	..	..	..	..	..	153
22	..	..	2	47	75	49	25	6	1	..	..	..	..	..	..	..	..	205
24	..	..	..	8	46	53	33	13	1	1	..	..	..	..	..	..	..	155
26	..	..	..	4	21	61	57	22	11	..	..	..	..	..	..	..	..	176
28	..	..	..	..	4	28	44	29	10	3	..	1	..	..	..	..	..	119
30	..	..	..	..	1	7	30	42	28	11	..	..	..	..	..	..	..	119
32	..	..	..	..	..	1	12	47	32	10	5	..	..	..	..	..	..	107
34	..	..	..	..	..	..	6	29	22	16	5	..	..	..	..	..	..	78
36	..	..	..	..	..	..	..	3	25	14	6	2	..	..	..	..	..	50
38	..	..	..	..	..	..	..	2	11	7	10	3	1	..	..	..	..	34
40	..	..	..	..	..	..	..	..	3	4	10	2	1	..	..	..	..	20
42	..	..	..	..	..	..	..	..	1	2	5	6	4	..	..	..	..	18
44	..	..	..	..	..	..	..	..	..	..	2	2	5	1	..	..	..	10
46	..	..	..	..	..	..	..	..	..	..	..	1	3	1	2	..	..	7
48	..	..	..	..	..	..	..	..	..	..	..	..	..	2	..	..	..	2
50	..	..	..	..	..	..	..	..	..	..	..	..	..	..	3	..	..	3
52	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	0
54	..	..	..	..	..	..	..	..	..	..	..	..	..	..	1	..	..	1
56	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	1	1
Total	5	25	105	196	236	254	220	195	145	68	43	17	14	4	6	0	1	1534

TABLE 16.—CORRELATION BETWEEN AVERAGE DAILY YIELD FOR SIXTH CALENDAR MONTH OF RECORD AND TOTAL YIELD FOR THE YEAR: GUERNSEY RECORDS

Yield for year, hundredweight F.C.M.: class mid-points

Average daily yield for the sixth month, pounds F.C.M. per day: class mid-points	55	65	75	85	95	105	115	125	135	145	155	165	175	185	195	205	215	Total
14	..	..	1	..	..	..	..	..	..	..	..	..	..	..	..	..	..	1
16	3	3	4	..	..	..	..	..	..	..	..	..	..	..	..	..	..	10
18	1	13	13	4	..	..	..	..	..	..	..	..	..	..	..	..	..	31
20	..	8	38	23	2	..	..	..	..	..	..	..	..	..	..	..	..	71
22	1	1	35	77	10	..	1	..	..	..	..	..	..	..	..	..	..	125
24	..	..	12	66	55	6	..	..	..	..	..	..	..	..	..	..	..	139
26	..	..	2	18	103	57	5	3	..	..	..	..	..	..	..	..	..	188
28	..	..	..	6	50	74	26	2	1	..	..	..	..	..	..	..	..	159
30	..	..	..	2	11	73	67	10	3	..	..	..	..	..	..	..	..	166
32	..	..	..	..	5	35	61	47	4	1	..	..	..	..	..	..	..	153
34	..	..	..	..	..	8	46	62	22	2	..	..	..	..	..	..	..	140
36	..	..	..	..	..	..	10	47	41	5	1	..	..	..	..	..	..	104
38	..	..	..	..	..	1	4	18	38	23	3	..	..	..	..	..	..	87
40	..	..	..	..	..	..	..	4	21	12	5	..	..	..	..	..	..	42
42	..	..	..	..	..	..	..	1	11	13	14	4	..	..	..	..	..	43
44	..	..	..	..	..	..	..	1	4	6	12	6	..	..	..	..	..	29
46	..	..	..	..	..	..	..	..	..	4	5	4	8	1	..	..	..	22
48	..	..	..	..	..	..	..	..	..	..	2	1	3	..	..	..	..	6
50	..	..	..	..	..	..	..	..	..	2	1	2	3	..	..	..	..	8
52	..	..	..	..	..	..	..	..	..	..	..	..	..	2	1	..	..	3
54	..	..	..	..	..	..	..	..	..	..	..	..	..	1	1	..	..	2
56	..	..	..	..	..	..	..	..	..	..	..	..	..	..	2	..	..	2
58	..	..	..	..	..	..	..	..	..	..	..	..	..	..	2	..	..	2
60	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	0
62	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	0
64	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	1	1
Total	5	25	105	196	236	254	220	195	145	68	43	17	14	4	6	0	1	1534

TABLE 17.—CONSTANTS FROM CORRELATION SURFACES FOR RATE OF YIELD AND YIELD FOR YEAR, ALL AGES: GUERNSEY RECORDS

Rate of yield and months after calving		Mean (pounds F.C.M.)	Standard deviation (pounds F.C.M.)	Coefficient of variability	Coefficient of correlation
Theoretical	0	39.20 ± .17	9.67 ± .12	24.66 ± .30	.672 ± .009
Theoretical	1	37.23 ± .15	8.58 ± .10	23.03 ± .28	.764 ± .007
Observed	1	36.88 ± .15	8.02 ± .11	21.74 ± .29	.767 ± .008
Theoretical	2	35.56 ± .13	7.77 ± .09	21.84 ± .27	.845 ± .005
Observed	2	35.82 ± .14	7.99 ± .10	22.31 ± .27	.827 ± .006
Theoretical	3	33.96 ± .12	7.22 ± .09	21.26 ± .26	.910 ± .003
Observed	3	34.06 ± .13	7.63 ± .09	22.40 ± .27	.874 ± .004
Theoretical	4	32.46 ± .12	6.83 ± .08	21.04 ± .26	.955 ± .002
Observed	4	32.58 ± .13	7.32 ± .09	22.48 ± .27	.914 ± .003
Theoretical	5	31.08 ± .12	6.72 ± .08	21.62 ± .26	.978 ± .001
Observed	5	31.35 ± .12	7.11 ± .09	22.68 ± .28	.929 ± .002
Theoretical	6	29.84 ± .11	6.60 ± .08	22.13 ± .27	.980 ± .001
Observed	6	30.17 ± .12	7.04 ± .09	23.34 ± .28	.935 ± .002
Theoretical	7	28.63 ± .11	6.65 ± .08	23.22 ± .28	.967 ± .001
Observed	7	29.03 ± .12	7.11 ± .09	24.48 ± .30	.931 ± .002
Theoretical	8	27.47 ± .12	6.74 ± .08	24.53 ± .30	.944 ± .002
Observed	8	27.86 ± .12	7.13 ± .09	25.58 ± .31	.914 ± .003
Theoretical	9	26.46 ± .12	6.91 ± .08	26.12 ± .32	.914 ± .003
Observed	9	26.83 ± .12	7.06 ± .09	26.33 ± .32	.884 ± .004
Theoretical	10	25.47 ± .12	7.09 ± .09	27.85 ± .34	.881 ± .004
Observed	10	25.93 ± .12	7.25 ± .09	27.97 ± .34	.858 ± .005
Observed yield for year		10 996 ± 41	2387 ± 29	21.71 ± .26	....

month is less closely related to the year's production. than is the calculated yield for that month. This is not surprising because of the considerable irregularities of the realized lactation curve. The condition is accentuated, furthermore, by the fact that the fat test for the month represents only two days, or one day in a minority of the records, thus increasing the variability of the F.C.M. observations. It is probable, in other words, that similar comparisons based on the raw milk yields would show higher correlations than those found for the F.C.M. values. One such correlation for raw milk yields has been computed for the 6th month and gives $r = .938 \pm .002$.

The correlation coefficients for the theoretical and observed lactation curves are given graphically in Fig. 18. It is evident that the correlations change in a quite regular manner with advance in lactation, reaching a maximum during the fifth month of lactation. There is a reason for this in terms of the theoretical lactation curves.

We have seen above (Fig. 4), with reference to the theoretical lactation curves, that the yields for the year are proportional to the ordinates of the curves at any specified time, provided k is constant. Theoretically, therefore, the only reason why there is not perfect correlation between the computed rate of yield and the observed yearly yield is because of variability in persistency. We must add to this reason the failure to adjust perfectly the theoretical lactation curves to the observed yields, and also some variability in the time after calving at which the record starts. The influence of variability in persistency, in affecting the correlation between the rate of yield at various times after calving and the total yield for twelve months, may be approached as follows:

Consider the lactation curve as represented by equation (1), $\frac{dy}{dt} = ae^{-kt}$, the area under which, from $t = 0$ to $t = 12$, representing the total yield for twelve months, is $\frac{a}{k}(1 - e^{-12k})$. It is clear from Fig. 5 that if we have given a point on the lactation curve at the origin, $t = 0$, the area under the curve will vary inversely with the value of k. On the other hand, if we have given a point on the lactation curve at the opposite end, $t = 12$, the area will vary directly with k. At some point in the curve between $t = 0$ and $t = 12$, the change in area with change in k passes from inverse to direct. Let this point be designated t'; that is, t' is an assumed point in the lactation curve where a change in k produces no change in area. Let $Y =$ the area, or twelve months' yield, $= \frac{a}{k}(1 - e^{-12k})$; and let $a' =$ the rate of yield at the time t', $= ae^{-kt'}$.

Then $a = a'e^{kt'}$ and $Y = \frac{a'e^{kt'}}{k}(1 - e^{-12k})$. We may determine the value of t' by differentiating Y with respect to k and setting $\frac{dY}{dk} = 0$, whence $\frac{dY}{dk} = \frac{a'e^{kt'}}{k} 12e^{-12k} + (1 - e^{-12k}) \frac{ka't'e^{kt'} - a'e^{kt'}}{k^2} =$

$$Y\left[t' - \frac{1}{k} + \frac{12e^{-12k}}{1 - e^{-12k}}\right] = 0, \text{ and } t' = \frac{1}{k} - \frac{12e^{-12k}}{1 - e^{-12k}}.$$

Solving for t' at the various class values of k found in the data, we have the results given in Table 18. It is evident that t' is not a fixed value but varies with the value of k. Applying the frequencies of Table 8 to the t' values of Table 18 leads to the constants: mean $= 5.476 \pm .006$; standard deviation $= .373 \pm .005$; coefficient of variability $= 6.81 \pm .08$.

For the k's actually found, therefore, there is a point in the lactation curves, $t = 5.476$, at which variation in k has little or no effect on the area. The rate of yield at this point should be directly proportional to the yield for the year regardless of the persistency value. By inference we accordingly may expect to find the highest degree of correlation between rate of yield and the yield for the year at or close to this point. This expectation is realized in the actual observations. Since the observed records start a few days after calving, there would be a tendency for the highest observed correlation to appear slightly later than the computed time.

TABLE 18.—SHOWING THE POINTS, t', IN THE LACTATION CURVES AT WHICH THE YIELD FOR THE YEAR IS NOT AFFECTED BY CHANGES IN PERSISTENCY, k

(The k values are the various class values represented in the Guernsey records)

$k \times 10^3$	−35	−25	−15	−5	5	15	25	35	45
t'	6.42	6.30	6.18	6.06	5.94	5.82	5.70	5.58	5.46
$k \times 10^3$	55	65	75	85	95	105	115	125	135
t'	5.35	5.23	5.11	5.00	4.88	4.77	4.66	4.55	4.45
$k \times 10^3$	145	155	165	175	185	195	205	215	....
t'	4.34	4.24	4.14	4.04	3.94	3.85	3.76	3.67	....

The constants of Table 17 afford the basis for estimating the yearly yield from the rate of yield per day. The equation from the smoothed lactation curves at six months after calving is $y = (354.40x + 421.8) \pm 318$; and from the actual yield of the sixth month $y = (317.02x + 1431.3) \pm 570$. The probable errors are for a single estimate.

Another point of interest in connection with Table 17 is in the mean values of the observed and calculated rates of yield. A fairly good

agreement is evident, but it appears that the k values have been slightly overestimated. That is, the calculated lactation curves average a slightly greater rate of decrease than is shown by the average actual data. Compare Fig. 6 in this connection.

Influence of Heredity and Environment.—It is well recognized by common observation that the yearly production of cows is influenced both by their ancestry and by the feed and care which they receive. Quantitative measures of the influence of heredity and environment on milk yield and fat percentage have been derived by statistical methods, and published (cf. particularly Gowen,[8] chap. 19). Similar methods are adopted here to study the relative effect of heredity and environment on persistency of lactation. Initial rate of yield, yield for the year, and fat percentage are also considered in the same connection as affording data of value and as a check on the results shown for persistency.

The records were sorted into herd groups by the owner's name. Herds were selected which contained at least two unrelated cows,[a] and at least two cows by the same sire and from different dams.

This condition was met by 72 herds containing 252 half-sisters by 97 sires, and 273 unrelated cows. The records of the cows with respect to persistency, initial rate of yield, yield for the year, and fat percentage have been correlated by the method of Harris.[12] The method is equivalent to using all possible combinations of the variables correlated, and gives a total number for half-sisters of 532; and for unrelated cows, 1,224. The derived constants are given in Table 19.

Corresponding correlations have also been derived for half-sisters from a common dam and by different sires; for full sisters; and for cows related as dam and daughter. The three combinations just mentioned were limited to the same herd, but on account of the limited number of such combinations all herds were included in which there were two or more cows of any of the relationships specified. The half-sister (common dam) combinations total 48; full-sister, 38; and dam-daughter, 54. The derived constants are given also in Table 19.

Since we are dealing now with smaller groups, there is more chance that any group may not be entirely representative. The values of the means given in Table 19 afford some indication as to this. It will be seen that there is fairly good agreement between the means for the several groups except in the case of the full-sister group. In this group both the initial rate of yield (A) and decrease in rate of yield (k) are high. Also the fat percentage of the full-sister group is distinctly above the average of all the records. There is consequently some question as to the representativeness of the full-sister group.

[a]By "unrelated" is meant that the cows were not related as half-sisters, full sisters or dams and daughters.

The coefficients of correlation given in the last section of Table 19 afford an indication of the relative influence of heredity and environment, within the range covered, on the performance of the cow. The coefficients for the unrelated herd mates show the degree of resemblance in performance due to similarity of environmental factors within a single herd and to dissimilarity of such factors as between the several herds represented. The resemblance may be due in part, also, to blood relationship more remote than that of half-sister.

Considering the data for fat percentage, which is well recognized as an individual and breed characteristic, it will be noted at once that daughters of a given sire within a particular herd tend to resemble each other more closely ($r = .305$) than do unrelated cows within a particular herd ($r = .138$). It will be recalled from the grouping used that each of the 72 herds is represented by at least two unrelated cows and by two half-sisters. It seems proper, therefore, to attribute the greater resemblance of the half-sisters to the additional effect of the common parentage on the sire's side.

A more definite measure of the influence of the sire may be obtained by the path coefficient method of analysis of Wright.[21] It may be assumed that the correlation between a half-sister and an unrelated herd mate would be the same as between unrelated herd mates. If we let 1 and 2 stand for related cows and 3 and 4 for unrelated cows, then with respect to fat percentage from Table 18, $r_{12} = .305$ and r_{34} ($= r_{13} = r_{23} = r_{14} = r_{24}$) $= .138$. The coefficient r_{12} is determined by the action of two forces: the common herd, or environment, and the common parentage, or heredity. Representing these forces by E and H respectively, the result is shown diagramatically in Fig. 19. The correlation between half-sisters is $r_{12} = .3054 = \mathrm{h}^2 + \mathrm{e}^2$; the correlation between unrelated cows, $r_{34} = .1376 = \mathrm{e}^2$. Hence $\mathrm{h}^2 = .3054 - .1376 = .1678$. The correlation between half-sisters with the herd influence eliminated, $r_{12 \cdot \mathrm{E}}$ becomes, $\dfrac{\mathrm{h}^2}{1 - \mathrm{e}^2} = \dfrac{.1678}{1 - .1376} = .1946$. This is the correlation to be expected between half-sisters on the sire's side within a single large herd or where the herd conditions are the same. Additional similar coefficients are given in Table 20.

It may be seen from Table 19 that half-sisters from a common dam show a similar degree of resemblance with respect to fat percentage as that shown by half-sisters by a common sire. The closer relationship, represented by full sisters or dam and daughter, shows a higher degree of resemblance.

TABLE 19.—STATISTICAL CONSTANTS OF CERTAIN CHARACTERS WITHIN GROUPS SELECTED ON THE BASIS OF BLOOD AND ENVIRONMENTAL RELATIONSHIP: GUERNSEY RECORDS

Intragroupal relationship		k × 10³ Persistency corrected for *A*	*A* Theoretical initial rate of yield, corrected for age (pounds F.C.M. per day)	Yield for year corrected for age (pounds F.C.M.)	Fat percentage for year
		Mean			
Unrelated herd mates[a]		39.62 ± 1.04	47.60 ± .40	12 943 ± 105	5.032 ± .021
Half-sister herd mates (C-S)[b]		41.03 ± 1.03	48.24 ± .41	13 008 ± 106	5.044 ± .020
Half-sister herd mates (C-D)[c]		40.71 ± 2.50	45.87 ± .78	12 775 ± 238	5.146 ± .043
Full-sister herd mates		47.97 ± 3.08	51.09 ± 1.18	12 741 ± 293	5.323 ± .052
Dam-daughter herd mates	dams	39.89 ± 2.42	46.29 ± .82	12 569 ± 237	5.116 ± .040
	daughters	42.37 ± 2.57	47.34 ± .82	12 943 ± 252	5.138 ± .044
		Standard deviation			
Unrelated herd mates		25.42 ± .73	9.75 ± .28	2583 ± 75	.514 ± .015
Half-sister herd mates (C-S)		24.29 ± .73	9.67 ± .29	2497 ± 75	.469 ± .014
Half-sister herd mates (C-D)		25.70 ± 1.77	8.04 ± .55	2447 ± 168	.445 ± .031
Full-sister herd mates		28.14 ± 2.18	10.82 ± .84	2681 ± 207	.479 ± .037
Dam-daughter herd mates	dams	26.42 ± 1.71	8.94 ± .58	2584 ± 168	.435 ± .028
	daughters	28.01 ± 1.82	8.91 ± .58	2743 ± 178	.474 ± .031

[a]Not related as dam and daughter, full sister, or half-sister. [b]Common sire, different dams. [c]Common dam, different sires.

TABLE 19—*Concluded*

Intragroupal Relationship	$k \times 10^3$ Persistency corrected for A	A Theoretical initial rate of yield, corrected for age (pounds F.C.M. per day	Yield for year corrected for age (pounds F.C.M.)	Fat Percentage for year
	Coefficients of variability			
Unrelated herd mates[a]	64.16 ± 1.85	20.48 ± .59	19.96 ± .58	10.21 ± .29
Half-sister herd mates (C-S)[b]	59.19 ± 1.78	20.06 ± .60	19.19 ± .58	9.30 ± .28
Half-sister herd mates (C-D)[c]	63.13 ± 4.35	17.52 ± 1.20	19.16 ± 1.32	8.64 ± .59
Full-sister herd mates	58.65 ± 4.54	21.18 ± 1.64	21.04 ± 1.63	8.99 ± .70
Dam-daughter herd mates { dams	66.23 ± 4.30	19.31 ± 1.25	20.56 ± 1.33	8.51 ± .55
Dam-daughter herd mates { daughters	66.11 ± 4.29	18.82 ± 1.22	21.19 ± 1.38	9.23 ± .60
	Coefficients of correlation			
Unrelated herd mates	.158 ± .019	.100 ± .019	.182 ± .019	.138 ± .019
Half-sister herd mates (C-S)	.157 ± .029	.267 ± .027	.405 ± .024	.305 ± .027
Half-sister herd mates (C-D)	.510 ± .072	.183 ± .094	.529 ± .070	.272 ± .090
Full-sister herd mates	.007 ± .109	.448 ± .087	.320 ± .098	.422 ± .090
Dam-daughter herd mates	.376 ± .079	.512 ± .068	.480 ± .071	.407 ± .077

[a]Not related as dam and daughter, full sister, or half-sister. [b]Common sire, different dams. [c]Common dam, different sires.

With respect to persistency of lactation it appears from Table 20 that there is no correlation between half-sisters by a common sire; but there is a considerable correlation between half-sisters from a common dam. There is also a significant correlation between dam and daughter.

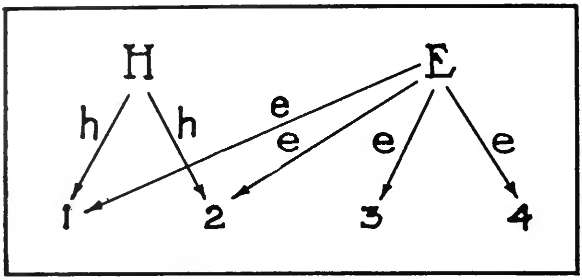


FIG. 19.—ANALYSIS OF THE CORRELATION BETWEEN RELATED HERD MATES, AS DETERMINED BY HEREDITY AND ENVIRONMENT

Unrelated herd mates are represented by 3, 4; related herd mates, by 1, 2. It is assumed that the correlation between unrelated herd mates (r_{34}) is the result of the common environment (E); the correlation between related herd mates (r_{12}) is the result of the common environment (E) *plus* the common parentage or heredity (H): $r_{34} = e^2$; $r_{12} = e^2 + h^2$; $r_{12 \cdot E} = \frac{h^2}{1 - e^2}$, the correlation associated with the blood relationship.

TABLE 20.—ESTIMATED COEFFICIENTS OF CORRELATION WITH RESPECT TO CERTAIN CHARACTERS, BETWEEN COWS OF VARIOUS BLOOD RELATIONSHIPS WITHIN AN INDEFINITELY LARGE HERD: GUERNSEY RECORDS

Relationship	Characters			
	Persistency of lactation corrected for *A*	Initial rate of yield corrected for age	Yield for year corrected for age	Fat percentage for year
	Estimated coefficients of correlation			
Half-sister, common sire	$-.001 \pm .029$	$.186 \pm .028$	$.273 \pm .027$	$.195 \pm .028$
Half-sister, common dam	$.418 \pm .080$	$.093 \pm .097$	$.424 \pm .080$	$.156 \pm .095$
Full-sister	$-.178 \pm .106$	$.387 \pm .093$	$.169 \pm .106$	$.329 \pm .098$
Dam-daughter	$.259 \pm .086$	$.458 \pm .073$	$.364 \pm .080$	$.313 \pm .083$

These relations suggest the interesting hypothesis that the persistency of a cow is affected by inheritance thru the dam but not thru the sire. The correlation between full sisters does not bear out such an hypothesis and in view of the large probable errors of the coefficients it may be questioned whether such an interpretation is justified. On the other hand, it has been noted above that the full-sister group does not conform closely to the average of all the records. Also, the correlation between full sisters with respect to yield for the year ($r = .169$) is less than is to

be expected from other results (Gowen,[8] p. 145). It seems clear that there is no resemblance between half-sisters on the sire's side with respect to persistency of lactation. This means either that the sire exerts no influence on his daughters or that all exert the same influence.

RESULTS FROM HOLSTEIN RECORDS

The Holstein data, in so far as they permit, will be presented in the same general fashion as has been used for the Guernsey records. The Holstein data do not include any record of the year's performance or date of freshening, so that relations involving these items cannot be considered. They do include one item not included in the Guernsey data, namely, the performance of the same cow in different lactations. There is, of course, a large amount of material of this kind available in the published Guernsey records, but the present study has been confined to original entries in the Guernsey records.

Age and Persistency.—The mean persistency values for the various age classes are given in Table 21 and Fig. 20. The mean values, like those for the Guernsey, are quite irregular, and a satisfactory representation by a smooth curve is difficult. It is clear, however, that there is a distinct rise in the curve at first, followed by a less rapid decline.

TABLE 21.—VARIOUS AGE CLASSES AND THE CORRESPONDING MEAN VALUES FOR PERSISTENCY AND THEORETICAL INITIAL RATE OF YIELD: HOLSTEIN RECORDS

Age in years (class mid-points)	Number of records	$k \times 10^3$ Persistency	A Initial rate of yield (pounds F.C.M. per week)
1.75	48	45.8	339.2
2.25	306	43.2	377.3
2.75	193	47.1	415.8
3.25	136	56.6	461.8
3.75	119	58.9	491.1
4.25	84	62.4	528.1
4.75	98	57.0	524.9
5.25	82	62.2	542.9
5.75	66	57.1	547.9
6.25	66	63.2	561.2
6.75	51	53.2	526.3
7.25	33	58.0	592.7
7.75	18	51.1	520.0
8.25	25	62.2	582.4
8.75	14	55.7	580.0
9.25	13	57.3	606.2
9.75	18	61.7	586.7
10.5	12	50.8	536.7
11.5	10	55.0	564.0
12.5	2	60.0	560.0
13.5	1	75.0	640.0

As compared with the Guernsey curve (Fig. 9), the changes with age are somewhat less pronounced.

Age and Initial Rate.—The mean values for initial rate of yield per week also are given in Table 21. Graphic presentation is given in Fig.

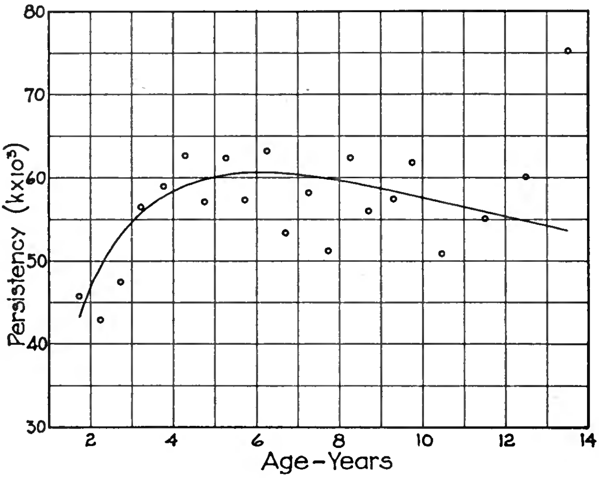


FIG. 20.—ILLUSTRATING THE CHANGE IN PERSISTENCY WITH ADVANCING AGE, HOLSTEIN RECORDS

Equation of the curve: $y = 35.6 - 8.36x + .167x^2 + 88.73 \log x$, x being the age in years. According to the equation, y reaches a maximum at $x = 6.09 = 6$ years, 1 month, 3 days, at which age $y = 60.5$. That is, the rate of decrease in the rate of yield per month at that age is 60.5 per mille per month.

21. The changes in rate of yield with age are quite regular up to 6½ years, after which there is considerable irregularity due in part to the small numbers represented. As compared with the similar data for Guernseys (Fig. 10), the changes with age are much more pronounced. It should be noted that the vertical scale of Fig. 21 is about half that of Fig. 10.

Persistency and Initial Rate.—The relation of persistency to initial rate of yield and to age may be shown approximately by dealing with the first part of the age curve in order to secure a reasonable approach to linear regression. For this purpose we may consider the records of those cows under 4½ years of age at calving. There are 886 such records, making up 63.5 percent of the total, and the constants are given in Tables 22 and 23.

Principal interest attaches to the correlation between persistency and age with initial rate constant, the partial coefficient of correlation

being .006. This agrees with the Guernsey results in indicating that persistency is independent of age except as the initial rate of yield is associated with age.

The correlation surface for persistency and initial rate of yield is given in Table 24 and the constants derived from this surface are pre-

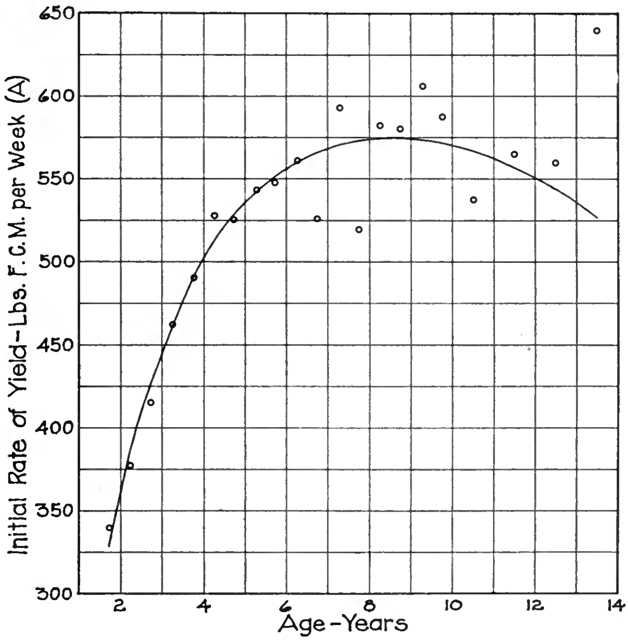


FIG. 21.—SHOWING THE CHANGE IN INITIAL RATE OF YIELD WITH ADVANCING AGE, HOLSTEIN RECORDS

Equation of the curve: $y = 213.5 - 23.50x - .546x^2 + 645.78 \log x$, x being age in years. This gives y a maximum value at $x = 8.542 = 8$ years, 6 months, 15 days, at which time $y = 574.4$, the maximum initial rate of yield in pounds F.C.M. per week.

sented in Table 25. The correlation ($r = .433$) is not quite as high as that found for the Guernsey records ($r = .535$). The mean persistency value, $k \times 10^3 = 53.2$, shows a considerably greater rate of decline than was shown by the Guernsey records. In this connection it is necessary to consider the initial rate of yield—which for the Holstein records is 471 pounds of 4-percent milk per week, or 67.3 pounds per day—as compared with 39.2 pounds of 4-percent milk per day for the Guernsey data. Thus while at first sight it might seem that the Holstein cows, as compared with the Guernsey, show a greater rate of de-

TABLE 22.—STATISTICAL CONSTANTS OF PERSISTENCY AND INITIAL RATE OF YIELD FOR AGES UNDER FOUR AND ONE-HALF YEARS: HOLSTEIN RECORDS

Constant	$k \times 10^3$ Persistency	A Initial rate (pounds F.C.M. per week)
Mean	50.2 ± .7	426.1 ± 2.3
Standard deviation	28.9 ± .5	99.3 ± 1.6
Coefficient of variability	57.58 ± .92	23.31 ± .37

TABLE 23.—SIMPLE AND PARTIAL COEFFICIENTS OF CORRELATION BETWEEN PERSISTENCY, INITIAL RATE OF YIELD, AND AGE: HOLSTEIN RECORDS UNDER FOUR AND ONE-HALF YEARS

Variables correlated	Variables constant	Coefficients
Persistency and initial rate	....	.407 ± .019
Persistency and age	....	.232 ± .021
Initial rate and age	....	.559 ± .016
Persistency and initial rate	Age	.344 ± .020
Persistency and age	Initial rate	.006 ± .023
Initial rate and age	Persistency	.523 ± .017

TABLE 24.—CORRELATION SURFACE FOR INITIAL RATE OF YIELD AND PERSISTENCY: HOLSTEIN RECORDS

Initial rate of yield, pounds F.C.M. per week (A): class mid-points

Persistency, rate of decrease per mille per month, $k \times 10^3$: class mid-points	240	280	320	360	400	440	480	520	560	600	640	680	720	760	800	840	880	Total
−25	1	1	..	..	1	..	..	..	..	..	..	..	..	..	..	..	..	3
−15	2	5	..	1	..	..	..	..	..	..	..	..	..	..	..	..	..	.. 8
− 5	..	7	3	3	2	1	1	1	..	..	..	..	..	..	..	..	..	18
5	7	10	7	6	11	2	1	4	..	..	..	..	..	..	..	..	..	48
15	1	5	17	12	13	8	6	5	5	..	1	..	..	..	..	..	..	73
25	5	15	23	24	16	15	15	15	3	5	3	..	4	..	..	..	..	143
35	1	8	17	22	18	30	20	14	6	10	4	1	1	1	..	..	..	153
45	..	5	16	26	29	38	30	29	10	5	6	7	5	2	..	..	..	208
55	..	2	6	23	31	24	31	29	20	13	10	4	7	3	2	..	..	205
65	..	3	4	14	18	20	21	23	17	22	8	3	4	2	1	1	..	161
75	..	1	3	6	10	17	25	22	18	16	16	7	2	2	1	..	..	146
85	..	2	5	4	8	19	15	15	11	13	8	2	2	..	3	1	1	109
95	..	..	..	..	3	6	9	12	10	5	3	3	3	1	..	..	..	55
105	..	..	..	1	1	2	5	7	3	4	2	1	2	..	1	..	..	29
115	..	..	..	2	1	1	..	2	3	2	1	..	..	2	..	1	..	15
125	..	..	..	1	3	..	2	2	1	..	..	1	1	..	..	..	..	11
135	..	..	..	1	..	..	..	..	..	..	..	3	..	..	..	..	..	4
145	..	..	..	..	..	..	..	..	1	..	..	1	..	..	..	..	..	2
155	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	0
165	..	..	..	..	..	..	..	1	1	..	..	..	..	..	..	..	..	2
175	..	..	..	1	..	..	..	1	..	..	..	..	..	..	..	..	..	2
Total	17	64	101	147	165	183	181	182	109	95	62	33	31	13	8	3	1	1395

crease in yield with advance in lactation, the very marked difference in level of production may require a modification of such a view.

The Holstein lactation curves, it will be remembered, are based on two 7-day tests; the second test occurring not less than eight months after calving. It would seem that there might be a very natural tendency to conduct this second test only on those cows which were milking heav-

ily enough to make a creditable record, that is, to exclude cows of more rapid decrease in rate of yield. Such a practice would tend to make the

TABLE 25.—STATISTICAL CONSTANTS FOR INITIAL RATE OF YIELD AND PERSISTENCY: HOLSTEIN RECORDS

Constant	$k \times 10^3$ Persistency	A Initial rate of yield (pounds F.C.M. per week)
Mean	53.2 ± .5	471 ± 2.1
Standard deviation	28.1 ± .4	116 ± 1.5
Coefficient of variability	52.85 ± .67	24.63 ± .31
Coefficient of correlation	.433 ± .015	

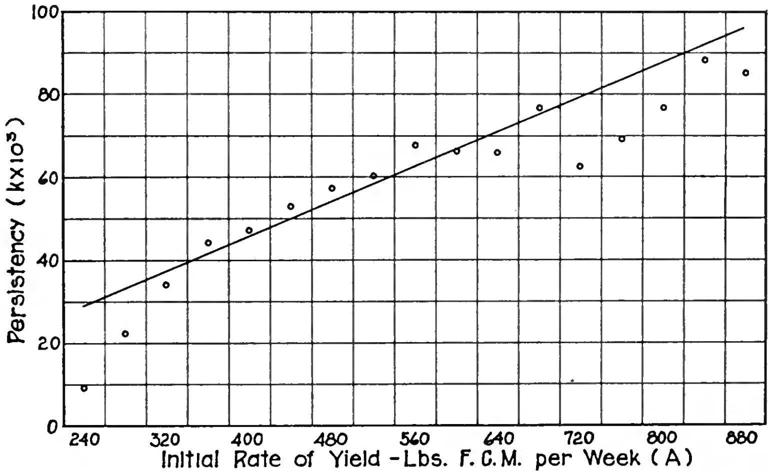


FIG. 22.—SHOWING REGRESSION OF PERSISTENCY VALUES ON INITIAL RATE OF YIELD, HOLSTEIN RECORDS

Equation of curve: $y = 3.75 + .105x$.

k value of the records too low to be representative. On the other hand, since few, if any, of the cows concerned were on yearly test, there would not be the incentive to the owner to maintain milk flow at the highest possible level after the first test. This condition would tend to give a higher value to k than might prevail for the same cows on yearly official test. Other possible complicating factors have been mentioned above (Fig. 7).

The regression of persistency on initial rate of yield is given in Fig. 22. The regression equation derived from the constants of Table 25 is $k \times 10^3 = 3.75 + .105A$. It will be seen from Fig. 22 that the mean persistency values at either extreme of the initial-rate range do not

conform very well with the regression curve. On the whole, however, a linear description appears to be justified for the bulk of the records. A noticeable difference between the Holstein and Guernsey data is in the slopes of the curves, 1.782 for the Guernsey and .735 (= .105 × 7) for the Holstein.

Correction Factors.—The initial rate of yield is corrected for age to the age of maximum initial rate as per the relations shown in Fig. 21.

A 20-inch slide rule provided with a specially graduated slide has been used in making these age corrections. The application of the method will be readily apparent. The proper multiplication factors at certain specified ages are derived from the equation. Graduations are made on the slide corresponding to these factors, but labeled with the appropriate *age* value instead of the value of the factor itself. This method of computation is highly advantageous in point of time and convenience. It also permits rather fine age distinctions (.1 month) at those ages, say under three years, where the factors are changing most rapidly. This feature practically offsets the four-figure limitation of legibility inherent in the dimension of the rule.

The persistency values are corrected for initial rate of yield to the mean initial rate, 741, according to Fig. 22, by the equation $k_c = k_o + 49.42 - .105A$; in which k_o is the observed value of $k \times 10^3$, and k_c the corresponding value corrected for initial rate of yield.

Variability in Persistency.—The distribution of the observed persistency values has been given in Table 24. Correction of these values for initial rate of yield leads to the distribution given in Table 26. The two distributions are presented graphically on a percentage basis in Fig. 23.

The corrected persistency values give a more regular distribution, a slightly higher mean, and considerably lower standard deviation as compared with the raw values. The coefficient of variability is very high in either case. When expressed in terms of the theoretical yields, the coefficient of variability takes a much more usual value, namely 13.05 (see last column of Table 26). The corresponding figure for the Guernsey records is 14.53. Hence it appears that there is only slightly less variability in persistency for the Holstein records than that found above for the Guernsey records.

Variability in Initial Rate of Yield.—The distribution with respect to initial rate of yield has been given in Table 24. Correction of these values for age gives the distribution presented in Table 27. The coefficient of variability is 18.84 and, as before mentioned, this value should be very closely related to the same constant for yield for the year. A noteworthy feature of these Holstein records is the high mean value of the

theoretical initial rate of yield, 571. This is equivalent to 81.6 pounds of 4-percent milk per day for the mature cow, or nearly double the comparable figure for the Guernsey records.

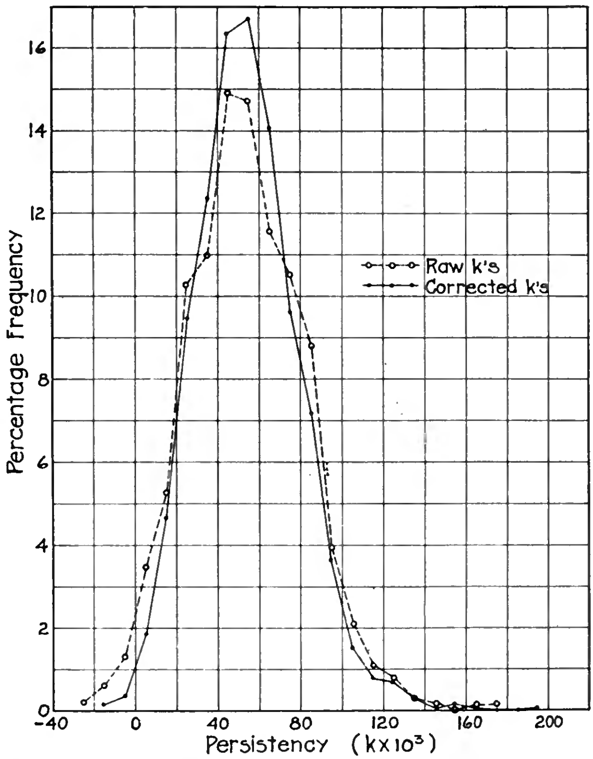


FIG. 23.—PERCENTAGE FREQUENCY DISTRIBUTIONS FOR PERSISTENCY OF LACTATION, HOLSTEIN RECORDS

The corrected *k's* have been corrected for initial rate of yield, as explained in text.

Rate of Yield and Yield for the Year.—It is of interest to determine for the Holstein records the point in the lactation curve that affords the best index of yield for a year. The records do not give the actual year's production, so that a direct comparison such as that made for the Guernsey records is not possible. It is possible, however, to consider the problem on a theoretical basis.

The points, t', on the lactation curves at which the area is unaffected by variation in k for the various persistency classes have been given in Table 18. The use of these values in conjunction with the persistency frequency distribution of Table 24 gives the following constants: mean

= 5.357 ± .005; standard deviation = .291 ± .004; coefficient of variability = 5.44 ± .07. From these results it may be assumed that the highest correlation between rate of yield and yield for the year would be found at about 5.36 months after calving. For any single random record the chances are even that the point t' would lie within about six

TABLE 26.—VARIABILITY IN CORRECTED PERSISTENCY VALUES AND CALCULATED YIELD FOR THE YEAR: HOLSTEIN RECORDS

(The persistency values have been corrected for initial rate of yield to A = 471.)

Frequency	$k \times 10^3$ Persistency corrected for A (Class mid-points)	Yield for year A = 471 (pounds F.C.M.)
2	− 15	26 982
5	− 5	25 380
26	5	23 902
65	15	22 537
132	25	21 275
172	35	20 110
228	45	19 028
233	55	18 029
196	65	17 099
134	75	16 237
100	85	15 439
51	95	14 694
21	105	14 000
11	115	13 356
10	125	12 754
4	135	12 194
1	145	11 669
2	155	11 178
1	165	10 721
0	175	......
0	185	......
1	195	9 510
Mean	54.23	18 327
Standard deviation	25.08	2 392
Coefficient of variability	46.25	13.05

TABLE 27.—VARIABILITY IN INITIAL RATE OF YIELD CORRECTED FOR AGE: HOLSTEIN RECORDS

(Mean = 571; standard deviation = 107.6; coefficient of variability = 18.84)

A[a]	308	336	364	392	420	448	476	504	532
f[b]	1	7	26	44	56	73	106	147	144
A	560	588	616	644	672	700	728	756	784
f	139	143	123	109	82	68	46	29	23
A	812	840	868	896	924	952	980	1008	1036
f	5	8	3	5	2	2	3	0	1

[a]Class mid-points of initial rate of yield in pounds of F.C.M. per week.
[b]Frequency.

days of this mean value. It is to be expected on the basis of the above results that the rate of yield during the fifth month of lactation would be fully as highly correlated with the comparable yield for the year in the case of these Holstein cows as has been shown to be the case for the Guernsey cows.

Influence of Heredity and Environment.—The comparisons made here are similar to those made above for the Guernsey records, with respect to persistency and initial rate of yield. Data for yearly yield and fat percentage for the year being lacking, the comparisons cannot be made for these items.

Those herds were selected in each of which there were at least two unrelated cows, as above defined, and also at least two half-sisters by a common sire and from different dams. The correlations with respect to persistency and initial rate of yield, between unrelated cows on the one hand and between half-sisters on the other hand, have been computed from the available records of the cows in these selected herds.

Similar correlations have been determined for half-sisters from a common dam and by different sires; for full sisters; for dam and daughter; and for different lactation records of the same cow. All herds which provided two or more records of any one of the relationships specified in this paragraph have been used in order to secure as large numbers as possible.

It may be assumed that any correlation between unrelated cows is due to the environmental relationship of being in the same herd. For the other groups there is added to this environmental relationship the blood relationship specified by the basis of the selection and, for the one case, the identity of the animal.

The number of herds used in the comparisons for unrelated cows and half-sisters by a common sire is 55; the number of combinations of unrelated cows, 1,620; the number of combinations of half-sisters by a common sire, 1,774; and the number of such common sires, 126. The number of combinations of half-sisters from a common dam is 78 and of full sisters, 64. There are 83 dam-daughter pairs, and 96 cows having two lactation records. The various statistical constants are given in Table 28.

The mean values of Table 28 show a general agreement between the several groups, altho with some points of diversity. It will be noted that the half-sister (common sire) herd mates and the unrelated herd mates agree quite closely in age-corrected initial rate of yield and also in persistency corrected for initial rate of yield. The daughters of the dam-daughter group show an exceptionally high rate of decrease in yield. The constants of the lactation curves indicate that the daughters of the dam-daughter group would have a somewhat lower yearly yield than their dams.

The initial rate of yield of the second records of the 96 cows in the last group show an increase of 48.8 pounds F.C.M. per week over the first records. The persistency values are practically the same. Evidently the correction factors have served to correct the indirect influence

TABLE 28.—STATISTICAL CONSTANTS OF CERTAIN CHARACTERS WITHIN GROUPS SELECTED ON THE BASIS OF BLOOD AND ENVIRONMENTAL RELATIONSHIP: HOLSTEIN RECORDS

Intragroupal relationship		$k \times 10^3$ Persistency corrected for A	Theoretical initial rate of yield corrected for age (pounds F.C.M. per week)
		Means	
Unrelated herd mates[a]		55.31 ± .43	574.6 ± 1.7
Half-sister herd mates (C-S)[b]		56.29 ± .40	580.4 ± 1.8
Half-sister herd mates (C-D)[c]		50.58 ± 1.04	609.3 ± 7.5
Full-sister herd mates		45.38 ± 1.42	582.2 ± 9.7
Dam-daughter herd mates	dams	49.57 ± 1.71	585.4 ± 7.0
	daughters	59.73 ± 2.16	585.2 ± 8.2
Same cow, same herd[d]	first records	48.45 ± 1.36	569.8 ± 7.1
	second records	48.73 ± 1.77	618.6 ± 7.2
		Standard deviations	
Unrelated herd mates		25.74 ± .31	99.9 ± 1.2
Half-sister herd mates (C-S)		25.22 ± .29	113.0 ± 1.3
Half-sister herd mates (C-D)		13.57 ± .73	97.6 ± 5.3
Full-sister herd mates		16.89 ± 1.01	115.6 ± 6.9
Dam-daugher herd mates.	dams	23.12 ± 1.21	94.3 ± 4.9
	daughters	29.14 ± 1.53	111.4 ± 5.8
Same cow, same herd	first records	19.74 ± .96	102.8 ± 5.0
	second records	25.65 ± 1.25	105.1 ± 5.1
		Coefficients of variability	
Unrelated herd mates		46.53 ± .55	17.38 ± .21
Half-sister herd mates (C-S)		44.81 ± .51	19.48 ± .22
Half-sister herd mates (C-D)		26.83 ± 1.45	16.02 ± .87
Full-sister herd mates		37.22 ± 2.22	19.86 ± 1.18
Dam-daughter herd mates	dams	46.64 ± 2.44	16.10 ± .84
	daughters	48.79 ± 2.55	19.04 ± 1.00
Same cow, same herd	first records	40.74 ± 1.98	18.05 ± .88
	second records	52.63 ± 2.56	16.99 ± .83
		Coefficients of correlation	
Unrelated herd mates		.144 ± .016	.064 ± .017
Half-sister herd mates (C-S)		.239 ± .015	.336 ± .014
Half-sister herd mates (C-D)		.478 ± .059	.261 ± .071
Full-sister herd mates		.067 ± .084	.367 ± .073
Dam-daughter herd mates		.462 ± .058	.329 ± .066
Same cow, same herd		.461 ± .054	.438 ± .056

[a]Not related as dam and daughter, full sister or half-sister. [b]Common sire, different dams. [c]Common dam, different sires. [d]Successive lactation records of the same cow.

of advance in age on persistency, but have not fully corrected the more direct influence of advance in age on initial rate of yield. Expressed in terms of yearly yield, the second record shows an increase in yield in excess of that expected on the basis of the average change in yield with age for the entire number (1,395) of cows. The excess increase amounts to about 8-percent of the first record. The most likely explanation for this excess increase is the artificial factor that there is no commercial object in conducting and reporting a second test on a cow unless that second test shows a relatively higher production than the first test.

TABLE 29.—ESTIMATED COEFFICIENTS OF CORRELATION WITH RESPECT TO PERSISTENCY AND INITIAL RATE OF YIELD, BETWEEN COWS OF VARIOUS BLOOD RELATIONSHIPS WITHIN AN INDEFINITELY LARGE HERD: HOLSTEIN RECORDS

Relationship	Characters	
	Persistency of lactation corrected for A	Initial rate of yield corrected for age
Half-sister, common sire	.111 ± .016	.290 ± .015
Half-sister, common dam	.390 ± .065	.211 ± .073
Full-sister	−.090 ± .084	.324 ± .075
Dam-daughter	.371 ± .064	.284 ± .068
Same cow[a]	.370 ± .059	.400 ± .058

[a]Correlation between successive lactation records of the same cow.

There is also the possibility of the excess increase being due to a physiological factor in the nature of an increased development of the mammary functions due to the exercise and training of the first test. The latter explanation has been noted by Gowen[9] and emphasized by Graves and Fohrman[10].

The coefficients of correlation show an appreciable resemblance between unrelated herd mates with respect to persistency and a smaller degree of resemblance with respect to initial rate of yield. The correlations for the various relationships within a single herd estimated by the method used above are given in Table 29.

These correlations with environment constant indicate a very slight relation between half-sisters by a common sire with respect to persistency of lactation, namely, r = .111. But the relation between half-sisters from a common dam is much more marked, r = .390. The relation between dam and daughter is of the same order, r = .371. And both these are slightly more closely related than are the first and second records of the same cow, r = .370.

The resemblance in persistency of cows related thru the dam appears to be much greater than those related thru the sire. This is in agreement with the Guernsey data. Also, like the Guernsey, the Holstein full sisters show no significant correlation in persistency. It is difficult to reconcile the results for full sisters with the results for other relationships, for full sisters have a common dam and it would seem should show as close a resemblance as half-sisters from a common dam.

While the small numbers reduce the significance of the correlations as a basis of generalization, the general agreement of the Guernsey and Holstein data make it appear that there is the possibility of a real difference in the correlation between the persistency values of cows according to their relationship thru the sire or thru the dam.

DISCUSSION

Selection of Records.—Certain of the Guernsey records, taken in the order of publication, have been excluded in the results as presented above. Only original entries have been used. A further selection was made on the basis of time of breeding, so as to eliminate the complication of the effect of advanced pregnancy on the lactation curve. The records showing very irregular lactation curves were also excluded. It is desirable to examine these records excluded in the above results.

An indication of the effect of the selection on the basis of time of breeding is afforded by the graphic presentation of Fig. 24. The data on which this figure is based are taken from Bulletin 272[6] and include reëntry records. There is considerable variability in the average initial rate of yield of the several groups separated according to the time of breeding (the farrow cows are shown as bred 12½ months after calving). There is a very slight tendency to a higher initial rate of yield in the groups in which breeding is delayed. On the other hand, there is quite a marked tendency to a slower rate of decrease in yield with the delayed breeding. The basis on which this rate of decrease has been determined practically eliminates the effect of pregnancy.

Delayed breeding is sometimes practiced to secure as large a record for the year as possible. The relations shown in Fig. 24 may be taken to mean that there is no particular tendency to select inherently higher yielding cows to be held open, but that there is a tendency to so manage the groups in which breeding is delayed as to cause them to hold up in milk flow better than the average. On this account the exclusion of records with a service period of less than six months tends to disturb the persistency distribution curve and to give a yield for the year somewhat higher than would hold for all the original entries.

The records which were excluded on account of the irregular nature of the lactation curve are treated in Fig. 25. The four individual lactation curves given serve to illustrate the diversity found as between individual records. The curve for No. 6356 shows an increasing rate of yield for five months, followed by a decreasing rate of yield; the curve for No. 7405 is roughly the inverse of that for No. 6356. The curve for No. 8032 shows a high rate of decline in approximate conformity with the equation type for six months, after which the rate of yield holds nearly constant; the curve for No. 8777 on the other hand, shows an increasing rate of decline during the last six months.

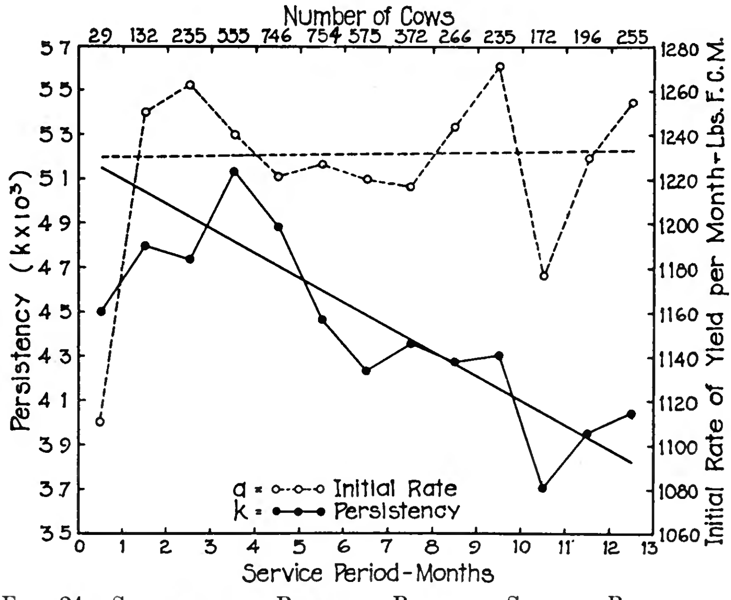


FIG. 24.—SHOWING THE RELATION BETWEEN SERVICE PERIOD AND INITIAL RATE OF YIELD AND PERSISTENCY, GUERNSEY RECORDS

Accepting the exponential curve as representing the normal course of affairs, it is apparent that the normal course is altered markedly in individual cases. When the individual curves of the irregular records (142) are thrown together and treated as a group, they fall in a fairly regular order (the circles in Fig. 25). Judged by the average curve alone, these irregular records conform very well to the exponential equation. The deviations of individuals are therefore of a compensating nature.

Chemical Interpretation of the Lactation Curve.[a]—Brody *et al*,[1] on

[a]Since this material was prepared, a somewhat more extensive treatment has been published by the author in an article, "Interpretation of the Lactation Curve," Jour. Gen. Physiol. 10, No. 1, 27-31. 1926.

the basis of the equivalence of the equations of the lactation curve and that of a monomolecular reaction, suggest that the rate of milk secretion is limited by such a chemical reaction. The heterogeneous nature of the performance of individual cows naturally raises some question as to the validity of such an interpretation.

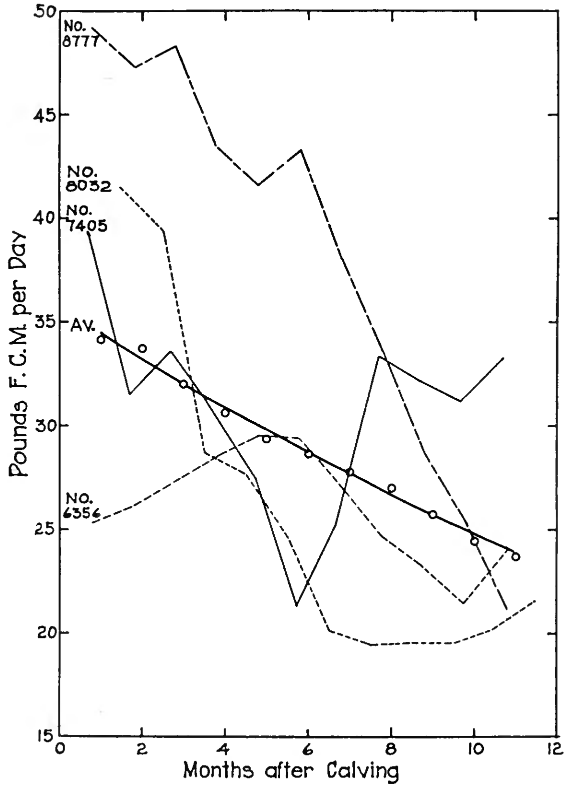


FIG. 25.—IRREGULAR LACTATION CURVES, GUERNSEY RECORDS

Four individual lactation curves (A.R. Nos. 6356, 7405, 8032, and 8777) are shown to illustrate the variation found in individual cases among the records excluded on the ground of the irregular nature of the lactation curves. The circles represent the average of the 142 records thus excluded. Equation of the corresponding smooth curve: $y = 35.78e^{-.03685t}$.

On the basis of such a chemical interpretation the rate of decline in milk yield is governed by the velocity constant, k of the equation. Tables 8 and 26 show a very high degree of variability in the constant k. It may be questioned whether the velocity constant of a particular chemical reaction, under constant temperature conditions, would show such a high degree of variability. The conception of a limiting reaction

of this kind furthermore is complicated by the fact that 5 percent of the individual records show an *increasing* rate of yield with advance in lactation for ten to twelve months. Apparently, if it is to be postulated that the rate of milk secretion is determined by a limiting substance, it must be further postulated that this limiting substance is not all present or active until some time (up to 12 months) after the beginning of lactation.

Breed Lactation Curves.—The initial-rate-of-yield distributions shown by the Guernsey records (Table 9) and by the Holstein records (Table 27) are brought together on a percentage basis in Fig. 26. There is apparent a very distinct difference in the two breeds with respect to the

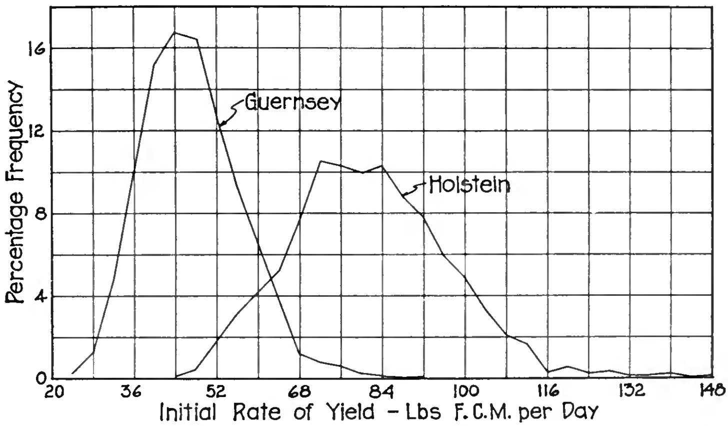


FIG. 26.—SHOWING PERCENTAGE FREQUENCY DISTRIBUTION CURVES OF THEORETICAL INITIAL RATE OF YIELD FOR GUERNSEY AND HOLSTEIN RECORDS

The data of both curves have been corrected for age to the age of maximum initial rate of yield.

A constant of the lactation curves. So far as indicated by the theoretical initial rate of yield, the Holstein records show a much higher productive capacity.

The persistency frequency distributions of the two breeds are shown in Fig. 27. The two curves in this instance are more nearly coincident but are not so directly comparable. Taken as they stand, they indicate that the Holstein records show a more rapid rate of decrease in rate of yield with advance in lactation than do the Guernsey records. The element that disturbs such a direct comparison is the fact that the persistency values have been corrected to a different initial rate base in the two cases.

The mean persistency value of the Guernsey records corrected to the mean initial rate is $k \times 10^3 = 44.81$ (Table 8). If correction were made

to the mean initial rate of the Holstein records by the equation for the Guernsey records, the mean persistency value for the Guernsey records would become $k \times 10^3 = 94.90$, as compared with $k \times 10^3 = 54.23$ for the Holstein records.

The mean persistency value of the Holstein records corrected to the mean initial rate is $k \times 10^3 = 54.23$ (Table 26). If correction were made to the mean initial rate of the Guernsey records by the equation of the Holstein records, the mean persistency value for the Holstein records would become $k \times 10^3 = 33.62$, as compared with $k \times 10^3 = 44.81$ for the Guernsey records.

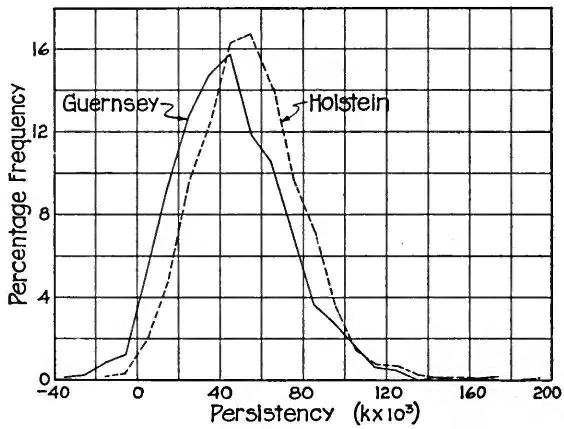


FIG. 27.—SHOWING PERCENTAGE FREQUENCY DISTRIBUTION CURVES OF PERSISTENCY OF LACTATION FOR GUERNSEY AND HOLSTEIN RECORDS

The persistency values refer to the rate of decrease per mille per month in the rate of yield of 4 percent milk per month. The data of both curves have been corrected for initial rate of yield to the mean initial rate of yield: namely, 39.2 pounds F.C.M. per day for the Guernsey records and 67.3 pounds F.C.M. per day for the Holstein records.

Hence it appears that while the Holstein records show a greater rate of decrease than the Guernsey records, yet when initial rate of yield is allowed for, the reverse is true. In view of the difference of the slope of the regression lines of persistency on initial rate of yield (Fig. 28), it is a question as to just how the two breed records may equitably be compared with respect to persistency, altho it would seem proper to say that the Holstein records show greater persistency (smaller value of k) than the Guernsey records.

Ellinger[3] mentions that the Red Danish breed is more persistent

than the Jersey breed. His measure of persistency is the ratio of the milk yield of the second 10-week period of the lactation to the milk yield of the first 10-week period. Possibly the breed differences in persistency are associated with size (weight), the larger cow tending to be more persistent than the smaller cow.

Carlyle and Woll[2] report results on persistency for cows in the experiment station herd classified in three groups: A, extreme dairy type (9 Jerseys, 4 Guernseys, 4 Holsteins); B, large dairy type (3 Jerseys, 5 Guernseys, 4 Holsteins); C, dual-purpose type (11 Shorthorns, 1 Red

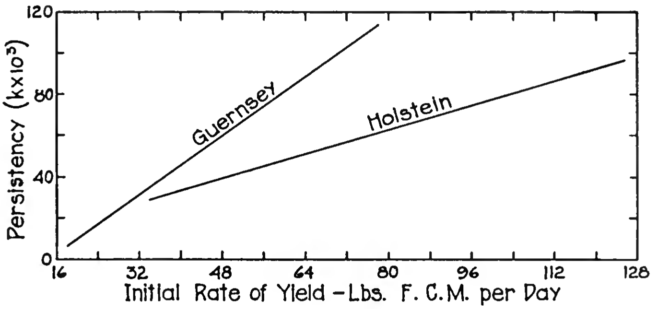


FIG. 28.—SHOWING THE REGRESSION OF PERSISTENCY OF LACTATION ON INITIAL RATE OF YIELD FOR GUERNSEY AND HOLSTEIN RECORDS

Polled). Their measure of persistency is the decrease in milk or fat yield of a late week in lactation as compared with an early week, the decrease being expressed as a percentage of the yield of the early week. Their results are somewhat variable, according to the particular two weeks of the lactation compared, but in general the extreme dairy type showed the smallest decrease, the large dairy type the largest decrease, and the dual-purpose type intermediate. These results are of interest in connection with the prevalent opinion as to the excellence of the dairy breeds in persistency of lactation, as compared with the dual-purpose breeds. It should be pointed out that the Wisconsin herd was probably composed of more or less select individuals and the same results might not hold for a larger and more representative population.

The breeding experiments of the Iowa Station are of interest at this point. The experiments referred to are based on the mating of dairy bred bulls with scrub cows. McCandlish *et al*[13] have studied the persistency of lactation of scrub cows, of purebred dairy cows, and of crossbred cows by dairy bulls from the scrub cows. They have measured persistency by expressing each month's milk yield as a percentage of the first month's milk yield. As thus expressed, the scrubs decrease most

rapidly, the dairy bred cows least rapidly, and the crossbreds are intermediate between the parent types. These results would indicate that there is a difference between dairybred cows and unimproved cows in respect to persistency of lactation. Correction of the persistency values for initial rate of yield would likely make the difference more pronounced. Apparently also the sire and dam influence the persistency of the offspring about equally.

Measures of Persistency.[a]—The measures of persistency mentioned above, it will be noted, are all based on a ratio of one sort or another. To these should be added the measure of Sanders,[16] $\frac{\text{total lactation yield}}{\text{maximum day's yield}}$. The length of the lactation period, and consequently the lactation yield, are greatly affected by the length of the service period. Sanders has therefore applied a correction factor to the ratios to reduce them to a standard service period. If yield for a definite time period were substituted for lactation yield in the ratio of Sanders, it is apparent the ratio would acquire a definite relation to the rate of decrease in yield. Turner[20] has presented a table for converting such ratios into the corresponding percentage decrease per month. It would seem preferable, in the use of such a system, to substitute a smoothed value of an initial period yield in place of the maximum yield. It is evident that the system applied literally would not distinguish between the performance of those cows which show an *increasing* rate of yield and those which show an approximately equal *decreasing* rate of yield.

Turner, in the paper mentioned, proposes also the method of dividing the yield for each calendar month by that of the preceding calendar month and using the arithmetic average of the ratios thus secured as a measure of persistency. It should be noted that such a method is mathematically unsound for the purpose in view, and where applied to the irregular data of individual records, tends to lead to too high results.

Correction Factors for Length of Record.—A set of correction factors for length of record from 200 to 365 days was presented in Bulletin 272,[6] based on an average persistency value for Guernsey records of $k \times 10^3 = 44.12$. It was pointed out that the factors would vary according to the values of k. The k frequency distribution (Fig. 14 and Table 6) gives an indication of the variability of k. It is apparent from the relation between k and yield ($r = -.226$) that the correction factors given are based on too high a value of k for low yields and too low a value of k for high yields. There is no simple way of taking account of

[a]Since this material was prepared, a somewhat more extensive treatment has been given by the author in a paper submitted to the Journal of Agricultural Research entitled, "Measures of Persistency of Lactation."

this known relation, but as the factors stand the error to which they are subject is not very serious so far as application to the Guernsey Advanced Registry records is concerned. It appears further that they should apply almost equally well to Holstein records, so far as may be judged by the present Holstein data.

Persistency as a Heritable Character.—Sanders,[16] from a study of the records of Shorthorn cows in an English milk recording society, came to the conclusion that the shape of the lactation curve, tho largely determined by environmental factors, is due partly to a genetic characteristic of the cow. His evidence for the genetic basis of persistency is the relation between the standard deviation of the $\frac{\text{total}}{\text{maximum}}$ ratios of individual records as compared with the standard deviation of individual mean ratios.

It is obvious that environment may be a large factor in determining persistency of lactation. Lactation may be terminated quickly at any stage by failure to remove the accumulated milk from the udder. It seems possible, therefore, that the character and frequency of milking may be factors in persistency. Obviously the feed supply is a very important factor and probably largely determines the difference between advanced registry practice, which results in an average rate of decrease of about 5 percent per month, and commercial practice, which results in an average rate of decrease of about 10 percent per month.

The data of McCandlish *et al*[13] presumably were obtained with environment constant, or nearly so, that is, within the same herd. The results seem to show unequivocally that dairy-bred cows are superior to scrub cows in persistency of lactation, and that the character is inherited apparently in a blending fashion. It may be noted incidentally that the lactation curve of the dairy-bred cows shows a rate of decrease of about 10.6 percent per month.

The results of the present paper indicate that persistency of lactation is nearly as definite an individual character as is initial rate of yield, judged by the fact that with the herd constant the correlation between two lactation records of the same cow is $r = .370$ for persistency and $r = .400$ for initial rate (Table 29). Neither of these coefficients is as high as similar comparisons reported by Gowen[8] for the Holstein breed in the case of yearly milk yield ($r = .667$) or fat percentage ($r = .715$). The effect of environment is not allowed for in the results of Gowen just quoted.

It is very clear that there is considerable variability between the individual records of cows with respect to persistency and, admitting that this is in part due to genetic differences of the individual cows con-

cerned, it follows that either the sire or the dam, or both, must have an influence on the persistency of the daughter. In the case of the Guernsey records it seems clear that as between the 97 sires studied there was no difference in their effect on the persistency of their daughters. Either the sire has no influence on the persistency of the daughter, or the 97 sires were all genetically alike with respect to this character. The latter alternative is forced by the results of McCandlish referred to above. While the Holstein records show a statistically significant correlation between half-sisters by a common sire and from different dams, the coefficient itself ($r = .111 \pm .016$) is too low to be of much practical importance. Obviously, if the differences in sires are not responsible for the inherent qualities of the daughters with respect to persistency, then differences in the dams must be responsible. Half-sisters from a common dam and by different sires, as well as daughter and dam, show a significant and material correlation, which is in accord with such an elimination conclusion. The anomalous results for full sisters on the other hand raise the question as to whether there is really any genetic difference between individuals within either of the two breeds with respect to persistency of lactation. That is to say, it may be possible that there is very little variability of a genetic nature among our present *dairy* breeds with respect to persistency of lactation.

The Short-Time Test.—Gavin[7] was perhaps the first to show definitely, by statistical treatment, the correlation between milk yield for a short period at the start of lactation and milk yield for the entire lactation. He found the correlation between lactation yield and yield for the 5th to 12th weeks to be $r = .858 \pm .005$, in the case of normal lactations 35 to 45 weeks in length. The corresponding relation for the maximum-days' yield was $r = .839 \pm .006$. The first correlation corresponds more or less closely in nature with the present correlation between the observed rate of yield two months after calving and yield for the year, shown in Table 17 to be $r = .827 \pm .006$. Gavin's data were taken from the herd records of Lord Rayleigh in England. It is evident from the coefficients of correlation that the present Guernsey Advanced Registry data are very similar to these English herd records with respect to the closeness of the relation under consideration.

Reference to Table 17 and Fig. 18 shows further that the correlation between rate of yield and yield for the year increases to $r = .935 \pm .002$ at six months after calving. This correlation would naturally be higher except for deviations of the observed lactation curve from the smoothed curve. The relation between the smoothed rate of yield and yearly yield is shown in Table 17 to be $r = .980 \pm .001$.

A question arises as to the records rejected because of unusual

irregularities. The correlation between the yield of the sixth month and the yearly yield for these 142 irregular records works out at $r = .839 \pm .017$, or materially lower than that found for the bulk of the records. Inclusion of these 142 irregular records with the other 1,534 records gives a correlation of $r = .928 \pm .002$.

Since the rate of yield is so closely related to the yearly yield it is a question why, in the interest of economy, a short-time test should not be used. A short-time test conducted in the fifth month of lactation affords a well nigh perfect index of the cow's 365-day energy yield under the conditions of the Guernsey Advanced Registry where pregnancy during the year period is eliminated as a factor. If the 365-day yield is the essential measure of a cow's performance, as seems to be assumed by the practice of the advanced registry system, why not use a six-months-after-calving short-time test to measure the cow's performance?

But the question should be asked, why select 365 days as a proper length of time to test a cow when that is only a part of her lactation period under the conditions that have developed in official testing? The selection of the 365-day period is undoubtedly connected with the economic significance of the year's yield under ordinary conditions of milk production, when the cows are bred to freshen once a year. The feed cost of milk production under such conditions bears a very definite relation to the yearly yield.

The relation depends upon the two main physiological facts that a certain quantity of nutrients is required for body maintenance and a certain further quantity is required for the elaboration of the milk itself. For the sake of simplicity we may consider a cow of 1,100 pounds live weight. According to Haecker,[11] the maintenance of an 1100-pound cow for one year requires 3,182 pounds of digestible nutrients, and the elaboration of one pound F.C.M. (4-percent milk) requires .343 pounds of digestible nutrients (cf. Gaines[4]). These quantitative relations lead to the equation $y = .343 + \frac{3182}{x}$, where y is the feed cost in pounds of nutrients per pound F.C.M. and x is the yearly yield in pounds F.C.M. The curve of this equation is given in Fig. 29. Cost in nutrients may be translated directly into cost in dollars, as has been shown by Ross *et al.*[15]

Fig. 29 shows clearly how the yearly yield is related to the cost of production, and why high yearly production is so important from the standpoint of efficiency of production. It should be emphasized that the relation between yield and cost as a generalization is based on a calving interval (time from one calving to the next calving) of one year.

The advanced registry record is often crowded past the point "X" of maximum efficiency of Fig. 29 and, also, usually fails to meet the calv-

ing interval condition mentioned above. While the selection of a year period as the time over which production is to be measured has a very fundamental basis in ordinary practice, the 365-day official record has

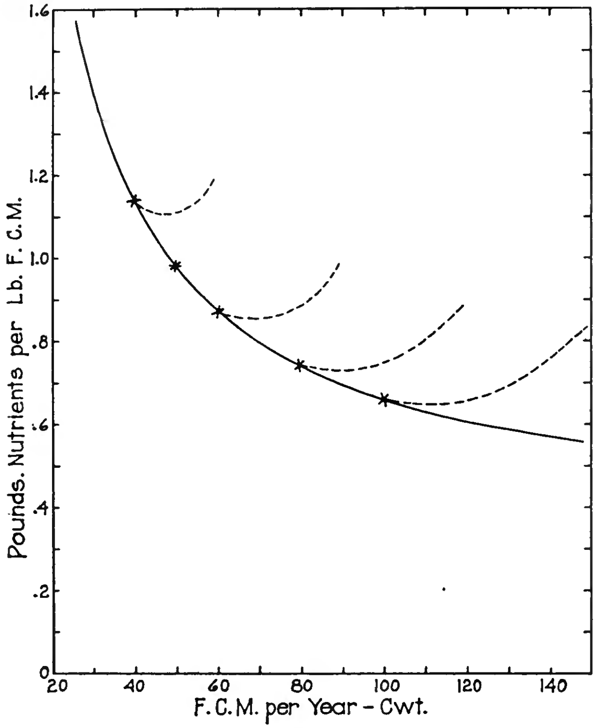


FIG. 29.—THEORETICAL RELATION OF YEARLY YIELD TO EFFICIENCY OF PRODUCTION

The solid curve represents the physiological limitation of efficiency of milk production by the 1100-pound cow, according to Haecker's feeding standard. The asterisk (*) on the curve shows the point at which is reached the often quoted efficiency of the cow, that is 18 percent of the feed nutrients recovered in the milk. The efficiency curve may be assumed to hold for any individual up to a certain point (X), beyond which some increase in yield is possible but at an extraordinary expense of nutrients and other costs, illustrated diagramatically by the broken curves. From the standpoint of economical milk production it is the point X on the yield curve that is of primary importance. The advanced registry record as a rule represents a point somewhere on the broken curve, often far past the point X. The broken curves are highly speculative.

developed into a state where the significance of the record from the standpoint of efficiency of production is very uncertain.

Fig. 30 presents a more or less diagramatic attempt to compare the advanced registry record with the ordinary record. The commercial

dairy-man has found by experience that the best practice is to breed his cows to freshen about once a year, regardless of the value of the calf at birth. The reason for this is to be found in the great natural stimulus of milk secretion associated with the reproductive process, and which

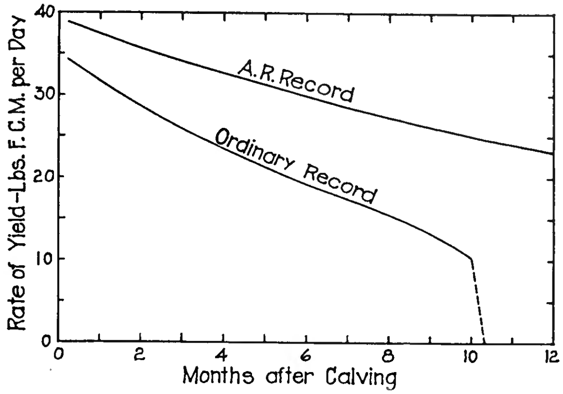


FIG. 30.—COMPARISON OF ADVANCED REGISTRY RECORD WITH ORDINARY RECORD

The A.R. curve is that of the present Guernsey records. As compared with the A.R. record, the ordinary record starts at a lower initial rate and declines at a more rapid rate. Furthermore, the cow is bred to freshen about once a year and allowed a dry or rest period before freshening. Frequent reproduction is essential to economical milk production, regardless of the value of the calf. It is the area under the lower type of curve on which the comparison of Fig. 29 is based. Yield for a year obtained from the upper type of curve cannot be used in the comparison of Fig. 29.

becomes apparent immediately following parturition. Practical experience has shown also that it is desirable to allow one to two months dry period before calving.

The importance of frequent reproduction may be demonstrated theoretically on the basis of the lactation curve. Assume[a] a dry period of about six weeks and a lactation yield in accordance with the general equation for a time two months less than the calving interval. Let c represent the calving interval in months, then the *average* yield per month for the interval is given by $a\dfrac{1 - e^{-k(c-2)}}{kc}$. It is the average yield over the calving interval that is economically important, and this may be computed in terms of a by assigning values to k and c. Table 30 gives certain results computed by this formula. Graphic presentation is given in Fig. 31.

It is apparent from Table 30 and Fig. 31 that, considering a persistency value of $k \times 10^3 = 100$, which is about what may be expected

[a]Since this paper was prepared the basis of the present assumption has been presented in more detail in a paper submitted by the author to the Journal of Dairy Science, entitled "Milk yield in relation to the recurrence of conception".

under ordinary conditions, the average yield per month or per year decreases quite rapidly as the calving interval increases. For example, consider a two-year period and an initial rate of yield of 1,000 pounds per month, cow A calving at the end of 12 months and again at 24 months,

TABLE 30.—AVERAGE YIELD PER MONTH FOR THE CALVING INTERVAL COMPUTED FROM THE LACTATION CURVE EQUATIONS

(Explanation in text)

Calving interval (months)	Average yield per month expressed as a percentage of the initial rate of yield per month			
	$k \times 10^3 = 25$	$k \times 10^3 = 50$	$k \times 10^3 = 100$	$k \times 10^3 = 150$
10	72.51	65.94	55.07	46.59
11	73.27	65.89	53.95	44.89
12	73.73	65.58	52.68	43.16
13	73.98	65.08	51.32	41.43
14	74.05	64.46	49.91	39.75
16	73.83	62.93	47.09	36.56
18	73.26	61.19	44.34	33.68
20	72.47	59.34	41.74	31.09
22	71.54	57.47	39.30	28.79
24	70.51	55.59	37.05	26.75

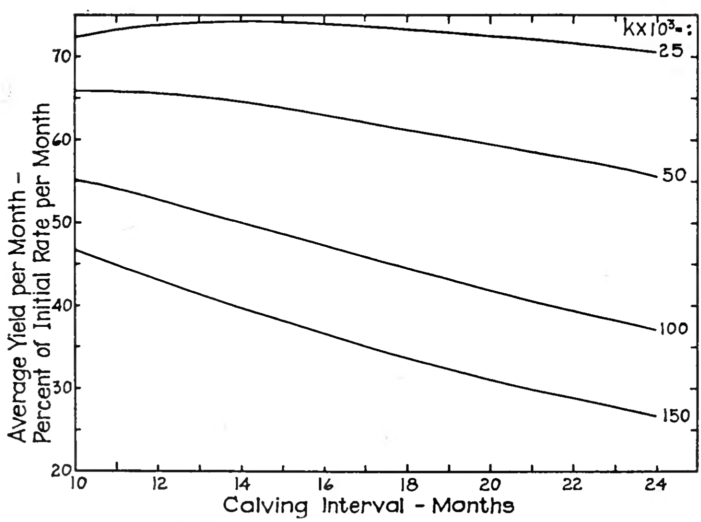


FIG. 31.—SHOWING THE NECESSITY OF FREQUENT REPRODUCTION FOR HIGH AVERAGE YIELD

It is assumed that milk is produced in accordance with the equation for a period two months less than the calving interval. The average yield, including both lactation and dry period, is greater the more frequent the breeding where $k \times 10^3$ is not less than 50.

cow B not freshening until 24 months. The two years' yield for A would be 12,643 [= 2(.5268 × 1000 × 12)] pounds while for B it would be only 8,892 (= .3705 × 1000 × 24) pounds. Furthermore, until the age of maturity the initial rate of yield ordinarily increases with each calf, which would give a still further advantage to cow A, breeding every 12 months. The illustration serves to show the great influence of frequent freshening on the average yearly yield over a series of years. The glamour of a large year-record representing only the first 365 days of a much longer lactation period should not obscure the economic issue.

In the economically practical type of lactation curve (ordinary record of Fig. 30), we have a lactation yield equal to, say, ten months' yield by the equation, and with a persistency value of about $k \times 10^3 = 100$. It becomes of interest to determine the point in the lactation curve at which the ten months' yield is unaffected by variability in persistency. The solution is given by the value of t' in the equation $t' = \frac{1}{k} - \frac{10e^{-10k}}{1 - e^{-10k}}$. For the value of $k \times 10^3 = 100$, $t' = 4.18$. That is, under the conditions of commercial practice a short-time test should afford the best measure of the cow's production if conducted at 4.2 months after calving.

The chief objection put forward to the short-time test is that it does not measure persistency of lactation and consequently does not accurately indicate yearly production. As a matter of fact, a short-time test at the beginning of lactation is a better measure of persistency than is the yield for a year. This statement is warranted by the fact that the correlation between theoretical initial rate of yield and persistency (rate of decrease) for the present Guernsey records is $r = .535$, whereas the correlation between yield for the year and persistency is only $r = -.226$. Yield for the year of course embodies the result of persistency, but any accurate measure of this character must be based directly on the lactation curve.

It is clear that if the short-time test is conducted at the stage of lactation where the yield for the portion of lactation under consideration is independent of variability in persistency, it becomes theoretically a perfect index of the yield for the longer period. For the purpose of representing the yearly production of the cow under ordinary conditions, bearing a calf yearly, it would seem that a short-time test conducted during the fourth month of lactation should afford a highly valuable measure of the cow's production from a practical standpoint. The yield for such a short-time test, like the yield for the year, would embody the result of persistency but would give no accurate measure of persistency.

As compared with initial rate of yield, or rate of yield at later stages of lactation, persistency is a much less important factor in determining the ordinary lactation yield. Differences in persistency are evidently very much subject to differences in factors of an environmental nature, and very little subject to differences in factors of a genetic nature, so far as indicated by the present records. From the inheritance standpoint, however, the present data are not extensive enough to establish satisfactorily the true condition of affairs, and judgment should be reserved pending investigation of a larger number of records.

On the other hand, as compared with persistency, initial rate of yield is a much more powerful factor determining the ordinary lactation yield; it is less subject to environmental factors; and it seems to promise considerable opportunity of improvement by selective breeding. Cows with inherent capacity for high yearly yield are clearly necessary to the most efficient production of milk. High yield for the year is to be attained only thru high initial rate of yield. Note, in Table 12, how distinctly the correlation surface of initial rate of yield and yield for the year is cut off at the upper right border.

It is a question for serious consideration whether we may not progress as well or better in breeding and selecting high-yielding cows on the basis of a short-time yield soon after calving, as we may on the basis of a short-time test later in the lactation, or on the basis of the lactation yield itself. Given high initial rate of yield, and regular, frequent reproduction, persistency of lactation seems to be of minor importance in the problem of breeding and selecting efficient cows.

SUMMARY AND CONCLUSIONS

A curve of the type $\frac{dy}{dt} = Ae^{-kt}$ has been fitted to each of 1,534 Guernsey records and 1,395 Holstein records. In the equation y is yield, $\frac{dy}{dt}$ is the rate of yield, A is the theoretical initial rate of yield, t is time from calving, and k is the rate of decrease in the rate of yield. The value of k is used as a measure of persistency of lactation. The value of A is representative of the rate of yield shortly after calving. The A and k constants of the individual curves have been studied statistically, and in case of the Guernsey records, the yield for the year and fat percentage for the year, also. Yield has been measured on an energy basis in terms of 4-percent milk.

It was found that A and k are quite closely related: for the Guernsey records $r = .535$; for the Holstein records $r = .433$. In both breeds A

increases with age to about nine years and then declines while k is independent of age with A constant.

Yearly yield is more closely related to A than to k: $r = .672$ and $r = -.226$ respectively. The correlation between yearly yield and rate of yield increases with advance in lactation up to six months, after which it decreases. At six months $r = .980$ for the smoothed lactation curve and $r = .935$ for the raw lactation curve. From the mathematical properties of the lactation curve, it seems that for the ordinary lactation of ten months and $k = .1$, the best time of conducting a short-time test to estimate the lactation yield is 4.2 months after calving.

The corrected persistency values show a high degree of variability, the coefficient of variability for the Guernsey and Holstein records being 60.97 and 46.25 respectively; and the corresponding coefficients for yearly yield as affected by persistency, 14.53 and 13.05.

The mean initial rate corrected to age of maximum is, for the Guernsey records, 47.2; and for the Holstein records, 81.6 pounds 4-percent milk per day. Corrected to the same initial rate, the Holstein records show greater persistency (lower value of k) than the Guernsey records.

Environment has a great effect on persistency. The evidence as to the influence of heredity on persistency with environment constant is inconclusive and conflicting: half-sisters by Guernsey sires show no correlation $r = -.001 \pm .029$; half-sisters by Holstein sires, a feeble correlation, $r = .111 \pm .016$. Half-sisters by a common dam show a material correlation, $r = .418 \pm .080$ (Guernsey) and $r = .390 \pm .065$ (Holstein). Dam-and-daughter relationships show $r = .259 \pm .086$ (Guernsey) and $r = .370 \pm .059$ (Holstein). Full sisters in both breeds show a non-significant negative correlation, $r = -.178 \pm .106$ (Guernsey) and $r = -.090 \pm .084$ (Holstein). Successive lactations of the same cow (Holstein) show a correlation of $r = .370 \pm .059$. All the correlations of this paragraph are for persistency corrected for initial rate of yield, and the correlations are corrected to eliminate the effect of common environment.

In view of the uncertainty of effecting further genetic improvement in our dairy breeds with respect to persistency of lactation, the low correlation between persistency and yearly yield, and the high correlation between rate of yield and yearly yield, it must be granted that a properly conducted short-time test may serve as an excellent production record, from both economical and biological standpoints.

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